OPTIMUM MACHINE SELECTION IN MULTISTAGE MANUFACTURING SYSTEMS

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Abstract

Machine selection during the initial stages of facility planning is critical as most capital costs are incurred in the investment of new machines. This paper presents an approach for optimizing the number of machines acquired for batch processing in a multistage manufacturing system. Machines of the same type differ in their processing capabilities. Four cases of overlap in machine capabilities are examined. For each case, the problem is optimized with and without the time balance constraints between the stages. The output is the optimum number of machines of each type to be purchased for each stage, as well as the optimum time delays between the stages that minimize total system cost.

Keywords: Multistage Manufacturing, Machine Selection, Optimization.

1. Introduction

A major cost component during initiation of a manufacturing plant is the capital investment in machinery and equipment. These investment decisions are critical to the profitability of the facility during its early stages of operation. A major decision involves the types and numbers of machines purchased. The types of machines selected depend on processing requirements of jobs that need to be performed. The number of machines of each type needed mainly depends on factors including the cost of machines, expected demand, and processing time.

Various researchers have investigated different methods for the machine selection problem. Most of the previous approaches can be classified into three categories: heuristic approaches, optimization techniques, or a hybrid of both methods. The development of heuristic rules is the most common approach used in industry. Subramaniam *et al.* (2000) propose three machine selection rules, which are tested by a simulator and found to improve scheduling performance. Taking the heuristic approach a step further, Chan *et al.* (2001) developed a knowledge-based expert system for machine selection of material handling equipment. Haddock and Hartshorn (1989) presented a decision support system to match part characteristics to machine specifications by taking into account criteria such as processing time and cost, machine availability, and their location. Although the heuristic approach has proven its feasibility through simulation, it nevertheless, presents a sub-optimal approach to the machine selection problem. Gutierrez and Sahinidis (1996) compared their optimization approach to machine selection with heuristic methods reported in the literature and found it to provide significant improvements.

When selecting machines for assigning jobs and tools, the total processing time is minimized (Abou Gamila and Motavalli 2003). Machine selection during initiation of a facility has been investigated (Jha and Jha 1998) where the optimum number of machines is determined for a new manufacturing facility. A hybrid approach proposed by Heragu and Gupta (1994) considers the design of cellular manufacturing facilities by developing a heuristic to optimize machine selection and other variables.

Most research in machine selection has focused on single machine problems. With the advent of computer-integrated manufacturing systems (CIM), the problem has been recently reconsidered for multimachine and multistage cases (Nagaraj and Selladurai 2002). The machine selection problem is considered for particular sets of job requirements in applications of CIM models in cellular manufacturing (Beaulieu *et al.* 1997, Chan *et al.* 1998), flexible manufacturing systems (Abou Gamila and Motavalli 2003), and just-in-time manufacturing (Gunasekaran *et al.* 1993). Gindy and Ratchev (1998) integrated machine selection in a decision-making environment. In this paper, we present a generalized procedure for the multistage machine selection problem in a batch processing facility. A mathematical model is developed to solve for the optimum number of machines of each type to be acquired for each stage. The model is solved by integer linear programming, exhaustive search, and branch and bound methods.

2. Problem Formulation

Figure 1 shows the multistage manufacturing system under consideration. It consists of p stages with q machines at each stage. The combinations of machines at each stage differ in their processing capabilities. A batch consisting of z parts arrives at the first stage. This batch is subdivided into m part groups by matching their processing requirements with the machine capabilities. The same part may appear in more than one group because they are matched to all applicable machines on which they can be processed. When the processing of a batch is completed at the first stage, it moves to the next stage and the parts are regrouped into new part groups according to the specified machine capabilities at this stage. All of the batches are identical in terms of part composition and numbers of parts. However, the part processing requirements vary from one stage to the next. So, this grouping procedure is repeated prior to processing at each new stage. The following assumptions are made in formulating the mathematical model:

1. The initial raw material stock for all parts to be manufactured is the same.
2. All parts may not require processing at every stage. However, all parts in the batch must move together from one stage to the next. Each stage is visited once and no backflow is allowed.
3. For a particular stage, the processing capabilities of the machines are not necessarily mutually exclusive. This means that there can be overlap in the capabilities of similar machine types. In addition, the same machine type may be duplicated in more than one stage.
4. A fixed time delay is imposed between consecutive stages on a balanced line and, in order to avoid accumulation of delays, the same time delay is also imposed between the first and last stages.
5. Annual demand, machine related costs, and penalty costs due to time delays are constant.

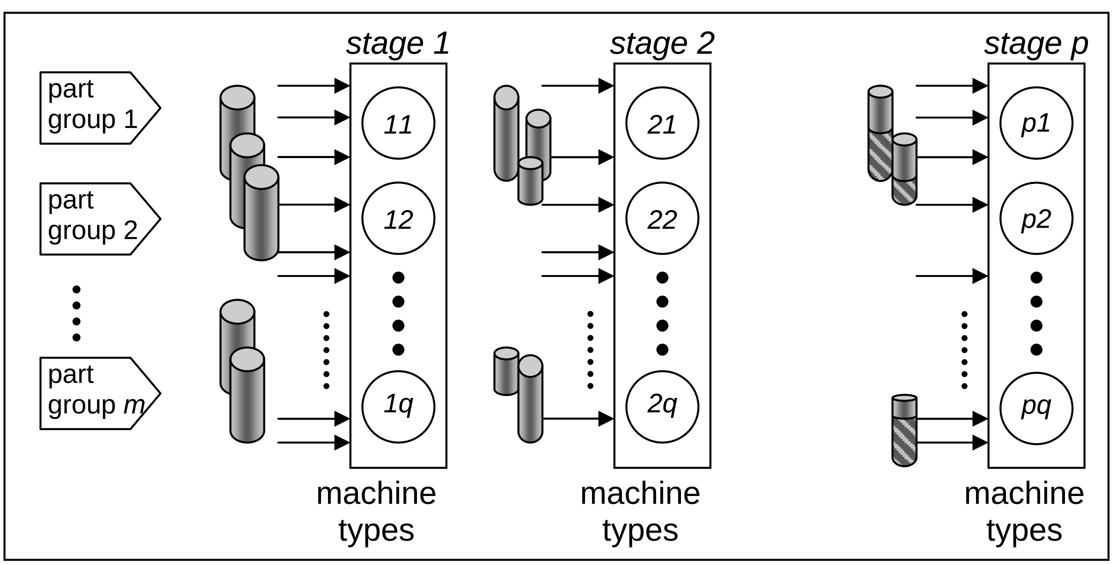

Figure 1. General configuration of a multi-stage production line.

Each machine type is characterized by its processing capability, its cost, and its effective capacity. The processing capability is defined by the dimensional limitations of the parts that can be handled by a particular machine. Each machine is associated with a cost coefficient that is used to represent the cutting costs involved. This includes the capital cost, operation costs, maintenance costs, etc. The actual cutting time available per period is defined as the effective capacity. The period is the processing time required per batch and it depends on total available time and demand.

The following notation is used in formulating the problem:

F = Cost function representing total system cost related to equipment selection.
i = Stage index $(i = 1, 2, \ldots, p)$.
j = Machine type index at each stage i $(j = 1, 2, \ldots, q_i)$, e.g. machine type can be drills.
k = Part number index $(k = 1, 2, \ldots, z)$.
g = Part group and machine group index at each stage i $(g = 1, 2, \ldots, m_i)$

where, m_i is the highest possible machine group index at each stage i, $m_i = \sum_{l=1}^{q_i} \binom{q_i}{l}$.

For the ith stage:

C_{ij} = Cost coefficient for machine type j at stage i in monetary units per period.
N_{ij} = Number of machines of type j at stage i (integer value).
$\beta_{i,i+1}$ = Penalty cost due to time delay between stages i and $(i+1)$ in monetary units per period.
$\varepsilon_{i,i+1}$ = Allowable time delay for the batch to move from stage i to stage $(i+1)$ where $\varepsilon \geq 0$.
E_{ij} = Effective capacity of machine type j at stage i in time units per period.
H_{ig} = Set of machines in a particular group, g, at stage i, $H_{ig} = \{1, 2, \ldots, m_i\}$.
r_{ig} = Set of part groups that can be processed by machine group g at stage i, excluding parts in group g, $r_{ig} = \{1, 2, \ldots, m_i\}$.
TA_{ig} = Total available processing time on machine group g at stage i; where, $TA_{ig} = \sum_{j \in H_{ig}} E_{ij} N_{ij}$.
t_{ig} = Processing time required by part group g on machine group g at stage i.
T_{ig} = Total processing time on machine group g at stage i where $T_{ig} = t_{ig} + \sum_{g \in r_{ig}} t_{ig}$ (note that a particular machine group may process several groups of parts).
$d_{i,i+1}$ = Maximum delay allowed between stage i and stage $(i+1)$; assumed constant for the production line.
Ω = Multiple of d representing the maximum total amount of delay allowed on the line.

The aim of the model presented below is to determine the optimum number N_{ij} of each machine type j needed at each stage i and to find the optimum delay $\varepsilon_{i,i+1}$ between the consecutive stages such that the line is balanced and the cost function F is minimized. Thus, N_{ij} and $\varepsilon_{i,i+1}$ are the design variables. The multistage line problem is formulated as follows:

Minimize

$$F = \sum_{i=1}^{p} \sum_{j=1}^{q_i} C_{ij} N_{ij} + \sum_{i=1}^{p-1} \beta_{i,i+1} \varepsilon_{i,i+1} + \beta_{1,p} \varepsilon_{1,p} \tag{1}$$

where $N_{ij} \geq 0$ *and integer for all i, j*
and $\varepsilon_{ij} \geq 0$ *for all i, j*

Subject to:

$$TA_{ig} \geq T_{ig} \qquad \text{for } i = 1,\dots, p \text{ and } g = 1,\dots, m_i \tag{2}$$

$$\left|TA_{ig} - TA_{(i+1)g}\right| - \varepsilon_{i,i+1} \leq 0 \qquad \text{for } i = 1,\dots, p \text{ and } g = 1,\dots, m_i \tag{3}$$

$$\varepsilon_{i,i+1} \leq d \qquad \text{for } 1 \leq i \leq p\text{-}1 \tag{4}$$

$$\sum_{i=1}^{p-1} \varepsilon_{i,i+1} \leq \Omega \qquad \text{for } i = 1,\dots, p \tag{5}$$

The complexity of the model is expressed in terms of the number of variables in the objective function and the number of constraint equations generated. The number of variables in the objective function consists of the total number of machines of type j, N_{ij}, and the allowable time delay, $\varepsilon_{i,i+1}$, for all p stages:

$$\text{Number of Variables} = \sum_{i=1}^{p} q_i + p$$

The number of constraints generated by equation (2) is the sum of the highest possible number of machine group combinations summed over all stages in the system. The number of constraints generated by equation (3) is equal to twice the number of stages because each absolute expression is linearized into two inequalities. Equation (4) represents the maximum allowed time delay between the stages, so the number of equations generated is one less than the number of stages in the system. Finally, equation (5) presents one constraint. In summary, the number of constraint equations generated in this multistage model depends on the numbers of stages and the highest possible number of machine groups formed. It is expressed as follows:

$$\text{Number of Contraints} = \sum_{i=1}^{p} m_i + 2p + (p-1) + 1 = \sum_{i=1}^{p} m_i + 3p$$

In the following subsections, brief explanations of the model components are presented.

2.1. *Modeling the objective function*

The first term in the objective function (1) represents the number of machines N of type j and their corresponding cost coefficients C at stage i. While the second and third terms in equation (1) consist of the product of the allowable time delay and the associated penalty cost. This delay applies to both the transition of the batch from one stage to the next, as well as the total time taken for the batch to be processed. It should be noted that the second term imposes the cost of delay between two consecutive stages, while the last term imposes an additional delay cost between the first and the last stage, p, to insure that delays between the nonadjacent stages do not exceed the allowable limit. For the special case of a two-stage line, where $p=2$, the last term is dropped from the objective function resulting in the following expression:

$$F = \sum_{i=1}^{p}\sum_{j=1}^{q_i} C_{ij} N_{ij} + \sum_{i=1}^{p-1} \beta_{i,i+1}\, \varepsilon_{i,i+1}$$

2.2. *Modeling the constraints*

Equation (2) represents the fact that the processing time cannot exceed the total available times on each machine group g at each stage i. The total available processing time is expressed in terms of effective capacity and number of machines as:

$$TA_{ig} = \sum_{j \in H_{ig}} E_{ij} N_{ij}$$

The right-hand side of equation (2) represents the total processing time T_{ig} and it is calculated as the time required for all parts to be processed by machines in group g and any of its subgroups that are capable of processing these parts. Therefore, $T_{ig} = t_{ig} + \sum_{g \in r_{ig}} t_{ig}$ where the first term represents the group that consists of all capable machines and the second term represents the sum of the processing time for various machine subgroups. Thus, equation (2) can be rewritten in terms of the optimization variables, N_{ij} as:

$$\sum_{j \in H_{ig}} E_{ij} N_{ij} \geq t_{ig} + \sum_{g \in r_{ig}} t_{ig}$$

Successive stages in the multistage manufacturing system are linked by a set of constraints represented by equations (3), (4), and (5). It is assumed that a fixed time delay is imposed between consecutive stages and between the first and last stage, in order to avoid accumulation of delays in a balanced line, as represented in equation (3). Equation (4) imposes a maximum delay d between the transitions of the batches from one stage to the next. The constraint represented in equation (5) prevents the overall delay from diverging in subsequent stages by restricting the sum of all delays to a constant Ω. For a two-stage line, Ω is the same as d; for a three-stage line, Ω=3d, which represents the delays between adjacent stages (1 and 2, 2 and 3) and between the first and the last stages, i.e., 1 and 3; and so on.

3. Case Examples

Two numerical examples are considered to illustrate and test the formulation. The first example is relatively small, which consists of three stages with two machine types per stage and five parts in each batch. The intention in presenting a small example is to illustrate the construction of the objective function and the generation of constraints in the model. The second example is a somewhat large and more realistic case, which consists of three stages with three machine types per stage and a batch size of 25 parts. It is used to illustrate the overall application of the optimization model and its results.

3.1. *Case example 1*

In this example, a three-stage production line is considered. Parts are processed on a lathe machine in the first stage, on a drill machine in the second stage, and again on a lathe machine in the third stage. After initial assessment of the process requirements, two types of lathe machines (*L1*, *L2*) are considered for the first and the third stages of processing and two types of drill presses (*D1*, *D2*) are considered for the second stage of processing. A batch size of five parts is processed in this production workshop.

Table 1 shows the required dimensions for the finished parts and the computed operation processing time for each part at each stage. The diameters are matched with machine capabilities to determine the part groups and the machines assigned. The processing time at each stage is calculated by using standard tool cutting time formulae for the following operating conditions: lathe operating at 338 rpm, 0.35 mm/rev feed, 2 mm maximum depth of cut, and cutting speed of 85 m/min; and a drill press running at 200 rpm and a feed rate of 0.08 mm/rev. For an annual demand of 17,500 batches, the time period is computed as 10 min/batch assuming the production system operates 365 days/year and 8 hours/day. Machine characteristics for this example are shown in Table 2.

Processing capability for the lathe is represented by the range of lengths that can be accommodated. As for the drill press, the range of hole diameters is the critical dimension by which its capability is determined. It is assumed that there is a 25% degree of overlap in the capabilities of similar type of machines. For example, if lathe 1 could process parts of length 0.1 to 0.5 m and lathe 2 could process parts of length of 0.5 to 0.9 m, they would have a 25% overlap. Maximum operation time taken by each part on a machine group is found by matching the machine capabilities with the processing requirements, as shown in Table 3.

Table 1. Machining data for case example 1.

	Part 1	*Part 2*	*Part 3*	*Part 4*	*Part 5*
Initial cylinder length, $X1_k$ (m)	0.77	0.49	0.28	0.62	0.41
Initial cylinder diameter (cm)	6.2	3.4	7.8	4.5	7.5
Final hole diameter, $X2_k$ (cm)	3.5	5.3	1.5	4	1.9
Final hole depth (cm)	5	6.5	3	4	5
Length, $X3_k$ (m)	0.77	0.49	0.28	0.62	0.41
Stage 1 – lathe time, $X4_k$ *(min)*	7	5	3	6	4
Stage 2 – drill time, $X5_k$ *(min)*	4	5	2	3	4
Stage 3 – lathe time, $X6_k$ *(min)*	4	3	2	3	2

Table 2. Machine characteristics for case example 1.

	Machine Type *(ij)*	Processing Capability	Effective Capacity, E_{ij} *(minutes/period)*	Cost Coefficient, C_{ij} *(cost /period)*
Stage 1	L1 (11)	$0.1 \leq X1 \leq 0.5$	6	40
	L2 (12)	$0.4 \leq X1 \leq 0.9$	8	60
Stage 2	D1 (21)	$0.25 \leq X2 \leq 4$	5	35
	D2 (22)	$3 \leq X2 \leq 7$	9	50
Stage 3	L1 (31)	$0.1 \leq X3 \leq 0.5$	6	40
	L2 (32)	$0.4 \leq X3 \leq 0.9$	8	60

Table 3. Operation time on machine groups for case example 1.

	Stage 1			*Stage 2*			*Stage 3*		
Part No.	L1 *only*	L2 *only*	*Both* L1, L2	D1 *only*	D2 *only*	*both* D1, D2	L1 *only*	L2 *only*	*both* L1, L2
1	0	7	0	0	0	4	0	4	0
2	0	0	5	0	5	0	0	0	3
3	3	0	0	2	0	0	2	0	0
4	0	6	0	0	0	3	0	3	0
5	0	0	4	4	0	0	0	0	2
Σ	3	13	9	6	5	7	2	7	5

For example, part number 1 has a dimension of 0.77m, which means it can be processed only on machine L2 in stage 1. The processing time required is 7 minutes as seen in Table 1. The remaining data in Table 3 are completed in a similar manner. The total time required for all parts on all possible machine group combinations are given in the last row of Table 3. A MATLAB program (Mathworks, Inc.) is written to generate the constraint set in this table automatically for all machine groups.

There are three machine and three part groups in each stage resulting in a total of 9 groups. Each column in Table 3 represents one machine group, while parts in each group can be seen by looking at the part/machine group combinations with non-zero values. For example, part number 3 makes one group; parts number 1 and 4 make one group; and parts number 2 and 5 make one group in the first stage. This grouping changes in the second stage. It can be seen that machine L1 shares 9 minutes of processing time with L2. Machine L1 has 3 minutes of its own workload while L2 has 13 minutes. These values represent the right-hand side of the constraint in equation (2). The maximum time delay d allowed between the stages is 5 minutes, therefore, Ω is 15 for this three-stage system.

It can be seen that machine L1 shares 9 minutes of processing time with L2. Machine L1 has 3 minutes of its own workload while L2 has 13 minutes. These values represent the right-hand side of the constraint in equation (2). The maximum time delay *d* allowed between the stages is 5 minutes, therefore, Ω is 15 for this three-stage system. The objective function and constraints for $\beta_{1,2} = 4$, $\beta_{2,3} = 3$ and $\beta_{1,3} = 4$ are defined as follows.

Minimize:

$$F = 40N_{11} + 60N_{12} + 35N_{21} + 50N_{22} + 40N_{31} + 60N_{32} + 4\,\varepsilon_{1,2} + 3\,\varepsilon_{2,3} + 4\,\varepsilon_{1,3}$$

Subject to:

(a) Processing time availability constraints, from equation (2):

$$6N_{11} \geq 3$$
$$8N_{12} \geq 13$$
$$6N_{11} + 8N_{12} \geq 25$$
$$5N_{21} \geq 6$$
$$9N_{22} \geq 5$$
$$5N_{21} + 9N_{22} \geq 18$$
$$6N_{31} \geq 2$$
$$8N_{32} \geq 7$$
$$6N_{31} + 8N_{32} \geq 14$$
$$N_{ij} \geq 0 \text{ for all } i, j$$

(b) Stage balancing constraints, from equation (3), with the absolute values linearized:

$$(6N_{11} + 8N_{12}) - (5N_{21} + 9N_{22}) - \varepsilon_{1,2} \leq 0$$
$$(5N_{21} + 9N_{22}) - (6N_{11} + 8N_{12}) - \varepsilon_{1,2} \leq 0$$
$$(6N_{31} + 8N_{32}) - (5N_{21} + 9N_{22}) - \varepsilon_{2,3} \leq 0$$
$$(5N_{21} + 9N_{22}) - (6N_{31} + 8N_{32}) - \varepsilon_{2,3} \leq 0$$
$$(6N_{11} + 8N_{12}) - (6N_{31} + 8N_{32}) - \varepsilon_{1,3} \leq 0$$
$$(6N_{31} + 8N_{32}) - (6N_{11} + 8N_{12}) - \varepsilon_{1,3} \leq 0$$

From equation (4):

$$0 \leq \varepsilon_{i,i+1} \leq 5 \quad \text{for } i = 1, 2$$

From equation (5):

$$\varepsilon_{1,2} + \varepsilon_{2,3} + \varepsilon_{1,3} \leq 15$$

A MATLAB program is written to automatically generate the constraints from the machining data and machine characteristics. The optimization is performed by linear programming (LP) using LINDO (Schrage, 1997); whereas exhaustive search (ES) and branch and bound (B&B) procedures are developed using MATLAB. The optimum numbers of machines of each type are feasible only if they are integers. Output of

LP and ES are non-integer and thus cannot be implemented in practice, unless they are rounded up to the nearest integer. However, the LP solution gives the lowest bound that can be achieved for the cost function and it is used for comparison purposes here. The LP results were used to approximate the lower limits for the maximum number of iterations of the design variables *N* and *ε* in the ES procedure. This is done to ensure that ES will terminate in a reasonable amount of time. The optimum solution is obtained by the B&B algorithm. The optimization was run for the following three cases:

1. Case without stage balancing constraints.
2. Case with stage balancing constraints and setting all delays between the stages $\varepsilon_{ij} \leq 5$ to ensure a reasonable value for the waiting time.
3. Case with stage balancing constraints and zero waiting time (i.e. all $\varepsilon_{ij} = 0$).

The results are presented in Table 4. Total cost was found to be highest when the waiting time between the stages was set to zero in a balanced system. This means that more machines must be acquired to eliminate any delays as the batches move from one stage to the next. The cost function is at its lowest when there are no balancing constraints or allowable delays. When balancing constraints are introduced with a maximum delay set to 5 time units between the stages, extra machines are required in some stages and the total cost increased from 420 to 564. When the time delay between the stages is restricted to zero, i.e., no delay case, the total cost increases further to 570 units with slight changes in machine requirements. This case example is presented to illustrate the development of the model, part/machine group formation, derivation of the objective function, constraint generation, and optimization outcome. In order to illustrate the applicability of the proposed model to real-life situations, a larger case is presented in example 2.

Table 4 Optimization results for case example 1.

	without balancing constraints		*with balancing constraints* ($\varepsilon_{ij} \leq 5$)		*with balancing constraints and zero waiting time* ($\varepsilon_{ij} = 0$)	
Design Variable	LP	B&B	LP	B&B	LP	B&B
N_{11}	2.00	2	2.00	2	2.00	2
N_{12}	1.63	2	1.63	2	1.63	2
N_{21}	1.20	2	1.20	2	1.20	2
N_{22}	1.33	1	1.56	2	2.11	2
N_{31}	1.17	1	2.17	3	3.00	2
N_{32}	0.88	1	0.88	1	0.88	2
$\varepsilon_{1,2}$	—	—	5.00	0	0	0
$\varepsilon_{2,3}$	—	—	0	2	0	0
$\varepsilon_{1,3}$	—	—	5.00	2	0	0
Cost Function, *F*	385.333	420	476.44	564	497.556	570

3.2. *Case Example 2*

A larger example with 25 parts, 6 machine types, and 3 stages is solved. Table 5 shows the machining data for each part in each batch. Since a large aspect of the solution depends on the amount of overlap with respect to machine capabilities, this case example was run for four different cases of overlap: (a) no overlap in machine capabilities, (b) 25% overlap, (c) 50% overlap, and (d) 100% overlap. Thus, we also illustrate here the effect of different processing capabilities by introducing different degrees of overlap as shown in Table 6. For instance, the part length processed by lathe L1 is between 0.1 and 0.3, then for the case of 25% overlap the range of length processed by lathe L2 is between 0.25 and 0.75 units. So for part number 3 that requires processing at a length of 0.29, it can be processed by either lathe L1 or lathe L2 since their processing capabilities overlap within this range. The case of 100% overlap means that there exists a universal machine, which covers the entire range of processing capabilities included in other machines in the same stage. The machining data in Table 5 and the machine characteristics in Table 6 are used to automatically generate the part/machine groups and constraints using the MATLAB program developed in this research. The formulation with the objective function and the constraints is constructed in a similar manner to case example 1.

Table 5 Machining data for a batch of products in case example 2.

	Stage 1 Turning		*Stage 2* Drilling		*Stage 3* Finishing	Machining Time		
Part No.	length, X_{1k} *(m)*	diameter *(cm)*	hole depth *(cm)*	hole diameter X_{2k} *(cm)*	length X_{3k} *(m)*	lathe time X_{4k} *(min)*	drill time X_{5k} *(min)*	lathe time X_{6k} *(min)*
1	0.48	18.66	18.66	9.33	0.48	5.00	6.00	3.00
2	0.77	16.38	16.38	8.19	0.77	7.00	6.00	4.00
3	0.29	10.92	10.92	5.46	0.29	3.00	4.00	2.00
4	0.25	5.19	5.19	2.60	0.25	3.00	2.00	2.00
5	0.22	16.84	16.84	8.42	0.22	2.00	6.00	1.00
6	0.63	14.03	14.03	7.02	0.63	6.00	5.00	3.00
7	0.22	16.60	16.60	8.30	0.22	2.00	6.00	1.00
8	0.18	8.39	8.39	4.20	0.18	2.00	3.00	1.00
9	0.45	6.17	6.17	3.08	0.45	4.00	2.00	2.00
10	0.13	4.44	4.44	2.22	0.13	2.00	2.00	1.00
11	0.11	5.16	5.16	2.58	0.11	1.00	2.00	1.00
12	0.00	0.00	18.73	9.37	0.00	0.00	6.00	0.00
13	0.24	19.98	0.00	0.00	0.24	2.00	0.00	1.00
14	0.56	19.43	0.00	0.00	0.56	5.00	0.00	3.00
15	0.56	5.55	5.55	2.78	0.56	5.00	2.00	3.00
16	0.00	0.00	6.80	3.40	0.00	0.00	3.00	0.00
17	0.55	17.69	17.69	8.85	0.55	5.00	6.00	3.00
18	0.52	4.75	4.75	2.37	0.52	5.00	2.00	3.00
19	0.74	14.42	14.42	7.21	0.74	7.00	5.00	4.00
20	0.54	13.85	13.85	6.93	0.54	5.00	5.00	3.00
21	0.42	13.10	13.10	6.55	0.42	4.00	5.00	2.00
22	0.63	17.28	17.28	8.64	0.63	6.00	6.00	3.00
23	0.81	8.80	8.80	4.40	0.81	7.00	3.00	4.00
24	0.31	17.59	17.59	8.79	0.31	3.00	6.00	2.00
25	0.14	17.96	17.96	8.98	0.14	2.00	6.00	1.00

Table 6 Machine characteristics for case example 2.

	Machine Type *(ij)*	Percentage Overlap				E_{ij}	C_{ij}
		None	25%	50%	100%		
Stage 1	L1 (11)	$0.1 \leq X1 < 0.3$	$0.1 \leq X1 < 0.3$	$0.1 \leq X1 < 0.3$	$0.1 \leq X1 < 0.3$	10	98
	L2 (12)	$0.3 \leq X1 < 0.7$	$0.25 \leq X1 < 0.75$	$0.2 \leq X1 < 0.8$	$0.1 \leq X1 < 0.9$	20	180
	L3 (13)	$0.7 \leq X1 < 0.9$	$0.7 \leq X1 < 0.9$	$0.7 \leq X1 < 0.9$	$0.7 \leq X1 < 0.9$	15	146
Stage 2	D1 (21)	$1 \leq X2 < 3$	$1 \leq X2 < 3$	$1 \leq X2 < 3$	$1 \leq X2 < 3$	10	96
	D2 (22)	$3 \leq X2 < 8$	$2.5 \leq X2 < 8.5$	$2 \leq X2 < 9$	$1 \leq X2 < 10$	20	160
	D3 (23)	$8 \leq X2 < 10$	$8 \leq X2 < 10$	$8 \leq X2 < 10$	$8 \leq X2 < 10$	15	139
Stage 3	F1 (31)	$0.1 \leq X3 < 0.3$	$0.1 \leq X3 < 0.3$	$0.1 \leq X3 < 0.3$	$0.1 \leq X3 < 0.3$	10	98
	F2 (32)	$0.3 \leq X3 < 0.7$	$0.25 \leq X3 < 0.75$	$0.2 \leq X3 < 0.8$	$0.1 \leq X3 < 0.9$	20	180
	F3 (33)	$0.7 \leq X3 < 0.9$	$0.7 \leq X3 < 0.9$	$0.7 \leq X3 < 0.9$	$0.7 \leq X3 < 0.9$	15	146

Table 7 shows the results obtained for the LP, ES, and B&B optimization for four cases of machine overlap. It is observed that the total cost decreases with increase in overlap. The LP solution is used as a benchmark. The ES procedure results in the minimum cost that can be achieved and it is used for comparison purposes. However, as the number of stages, numbers of machines in each stage, and the number of parts are increased, then the exhaustive search requires excessive computation time that prohibits it from becoming a viable solution method. Branch and bound method, which gives almost the same results as exhaustive search, is the fastest method to use for these types of problems. The branch and bounding procedure in the B&B implemented is done automatically in LINDO. Table 8 presents the solution for the case with balancing constraints when the problem was solved with the same balance delay cost coefficients of $\beta_{1,2} = 4$, $\beta_{2,3} = 3$ and $\beta_{1,3} = 4$. The maximum allowable time delay between stages is set to 7 time units.

The total cost figures have all increased, as compared to those in Table 7, due to introduction of additional constraints resulting from balancing requirements between the stages. This is because more machines are required in some stages to ensure that the stage balancing requirements are met. The same trend of reduced cost with increased machine overlap is observed in both cases, with and without balance constraints. The balance delays ε are also shown in Table 8. In the case of no overlap, B&B solution results in $\varepsilon_{1,2}=0$, $\varepsilon_{2,3}=5$, and $\varepsilon_{1,3}=5$. It means that each batch will be delayed by 5 time units between stages 2 and 3; and no delays between stages 1 and 2.

Table 9 summarizes the results for various cases of machine overlaps with no wait time. In this case, it is assumed that the balance delay between the stages would be 0 for all stage combinations. As can be seen from the table, the optimum numbers of machines required to achieve the demand per period are higher than in previous cases. The cost is also much higher with the same decreasing tendency as the overlap capabilities are increased. This is expected since tighter constraints are introduced to balance the line with zero delays between the stages.

Table 7 Optimization results for case example 2 – without balancing constraints.

	no overlap			*25% overlap*			*50% overlap*			*100% overlap*		
Design Variable	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B
N_{11}	1.90	2	2	1.30	2	2	0.70	2	2	0	0	0
N_{12}	2.65	3	3	3.30	3	3	3.95	3	3	4.65	4	4
N_{13}	1.40	2	2	0.93	1	1	0.47	1	1	0	1	1
N_{21}	1.00	1	1	0.40	1	1	0	1	1	0	0	0
N_{22}	1.75	2	2	2.95	2	2	4.35	4	3	4.95	5	5
N_{23}	3.60	4	4	2.40	4	4	0.80	1	2	0	0	0
N_{31}	1.10	2	2	0.70	1	1	0.40	1	1	0	0	0
N_{32}	1.50	2	2	1.90	2	2	2.25	2	2	2.65	3	3
N_{33}	0.80	1	1	0.53	1	1	0.27	1	1	0	0	0
Cost Function, F	2238.6	2702	2702	2190.1	2458	2458	2138.1	2361	2340	2106	2206	2206

Table 8 Optimization results for case example 2 – with balancing constraints.

	no overlap			*25% overlap*			*50% overlap*			*100% overlap*		
machine type, (*ij*)	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B
N_{11}	1.90	2	2	1.30	3	3	0.70	1	2	0	0	0
N_{12}	2.65	3	3	3.30	3	3	3.95	4	3	4.65	4	4
N_{13}	1.40	2	2	0.93	1	1	0.47	1	1	0	1	1
N_{21}	1.00	1	1	0.40	1	1	0	1	1	0	0	0
N_{22}	1.75	2	2	2.95	2	2	4.35	4	3	4.95	5	5
N_{23}	3.60	4	4	2.40	4	4	0.80	1	2	0	0	0
N_{31}	1.10	2	3	0.70	1	1	0.40	1	2	0	0	0
N_{32}	3.45	3	3	3.85	4	4	4.20	4	3	4.60	4	4
N_{33}	0.80	2	1	0.53	1	1	0.27	1	1	0	1	1
$\varepsilon_{1,2}$	6	5	0	6	6	5	6	5	5	6	5	5
$\varepsilon_{2,3}$	7	7	5	7	7	5	7	7	5	7	7	5
$\varepsilon_{1,3}$	1	0	5	1	0	0	1	0	0	1	0	0
Cost Function, F	2638.6	3069	3015	2590.1	2961	2951	2538.1	2844	2563	2506	2573	2567

Table 9 Optimization results for case example 2 – with balancing constraints and no wait time.

	no overlap			*25% overlap*			*50% overlap*			*100% overlap*		
machine type, (*ij*)	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B	LP	ES	B&B
N_{11}	1.90	2	2	1.30	2	2	0	0	0	0.70	1	1
N_{12}	2.95	3	3	3.60	4	3	4.95	5	5	4.25	4	4
N_{13}	1.40	2	2	0.93	1	2	0	0	0	0.47	1	1
N_{21}	1.00	1	1	0.40	1	1	0	0	0	0	1	1
N_{22}	1.75	2	2	2.95	3	2	4.95	5	5	4.35	4	4
N_{23}	3.60	4	4	2.40	3	4	0	0	0	0.80	1	1
N_{31}	1.10	2	2	0.70	2	2	0	0	0	0.40	1	1
N_{32}	3.80	3	3	4.20	4	3	4.95	5	5	4.55	4	4
N_{33}	0.80	2	2	0.53	1	2	0	0	0	0.27	1	1
Cost Function, F	2706.6	3028	3028	2658.1	3117	3028	2574	2600	2600	2606.1	2803	2803

4. Concluding Remarks

This paper presents a generalized procedure for machine selection in a multistage manufacturing facility with batch processing. Balanced and unbalanced cases were considered in addition to machine overlap capabilities. The unbalanced system has a smaller total machine cost than the balanced one, since more machines are needed in some stages to achieve a required balance. The possible reduction in total cost due to improved production rate of a balanced line was not included in the model. Total cost was the highest when the waiting time between the stages was fixed to be zero in the balanced system, which means more machines were acquired to eliminate any time difference between the stages. As the machine overlap capabilities increased, the cost decreased. This is because, with overlapping some machines may do the job of other machines and thus a better distribution of the parts is achieved on the machines, resulting in higher machine utilizations, less numbers of machines, and lower total cost.

The total cost function can be minimized by changing the part properties, by changing the design of the parts, which may lead to some changes in the constraints, by introducing better machine capabilities, and by reducing machine costs. For an effective production system, total costs must be minimized. Therefore, machine selection problem should be carefully formulated and converted into a suitable mathematical model. An appropriate algorithm, as outlined in this paper, should be utilized to solve the problem.

Possible extensions to this work include the consideration of backflow of parts. This would result in a different grouping of parts and, therefore, affect the processing time required. The model for the multistage system is developed for a linear machine layout case. Alternative layouts could be considered, such as circular and loop layout. In these cases, the allowable time delay should take into account the distances, hence time, that the parts move between various machines (intracell) in the same stage, in addition to the inter-cell distances moved between the stages. Another case to consider would be a continuous, rather than a batch, production system.

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