DEVELOPMENT OF A PRODUCTION PLANNING MODEL FOR PROCESS INDUSTRY ENVIRONMENTS

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Abstract: Process industries generally feature specific aspects such as continuous production of standardized goods in large volumes, the use of expensive specialized equipment and the focus on capacity utilization etc. In this study, we consider the multi item capacitated lot sizing and sequencing problem in process industry-type environments featuring intensively time consuming and sequence dependent setups, which necessitate the integration of the lot sizing and sequencing steps in the production plan. Formulating a nonlinear mathematical model to decompose the problem into two parts for lot sizing and sequencing respectively, we propose an iterative procedure for the solution.

Keywords: *Production Planning, Sequence-Dependent Setups, Mathematical Programming*

PROSES ENDÜSTRİSİ TİPİ ORTAMLAR İÇİN BİR ÜRETİM PLANLAMA MODELİNİN GELİŞTİRİLMESİ

Özet: Proses endüstrileri, büyük miktarda standart ürün üretimi, pahalı özel ekipman kullanımı, kapasitenin verimli kullanımına odaklanma gibi özellikler taşır. Bu çalışmada, sıraya bağlı ve uzun süren hazırlıklar içeren, çok ürünlü ve kapasite kısıtlı proses endüstrisi tipi ortamlarda parti büyüklüğü belirleme ve sıralama problemi ele alınmıştır. Bu problem, parti büyüklüğü belirleme ve sıralama aşamalarının beraberce düşünülmesini gerektirmektedir. Problem, doğrusal olmayan bir model formulasyonü geliştirilerek iki kısma ayrıştırılmış ve çözüm için iteratif bir yöntem önerilmiştir.

Anahtar Kelimeler: *Üretim Planlama, Sıraya Bağlı Hazırlıklar, Matematiksel Programlama*

1. Introduction

Capacitated lot sizing problems have been the subject of innumerable studies in the mathematical programming literature. In the case where sequence-dependent setups consume an important portion of the available capacity in a period, lot sizing and sequencing decisions need to be made simultaneously, unlike typical production planning practice where these two decisions are sequentially made. As such, even finding a feasible solution to the problem becomes extremely difficult. Our main motivation in this study is to develop an integrated mathematical model for capital-intensive process industry environments, of which the particular setting described above is highly representative.

The paper describing our work up to present is organized as follows: In Section 2, we present the main GLSP (General Lotsizing & Scheduling Problem) formulation. Section 3 depicts our mathematical programming based solution approach, which constitutes the decomposition of the problem into relatively easier sub-models, and an iterative solution procedure. Section 4 concludes the discussion with references to the study currently going on and its possible future directions.

2. Problem Formulation

Our main formulation is based on the GLSP, which is an integrated single-level, multi-item capacitated lot sizing and sequencing model with sequence dependent setup costs and times. All model parameters and decision variables are presented in Table 1 on the following page along with the mathematical model.

This model determines the production quantity of items in positions through the time periods. An item can be produced more than once within a period. Two particular aspects special to our formulation are the existence of setup times and the possibility of overtime in a period, compared with earlier GLSP formulations in the literature (Drexl and Kimms (1997), Fleischmann and Meyr (1997)).

Table 1. Parameters and Decision Variables for the GLSP Model

Parameters	Decision Variables
ST_{ij}: Setup time for the transition from item i to item j	X_{jn}: Production quantity of item j in position n
SC_{ij}: Setup cost for the transition from item i to item j	W_{jn}: Binary variable indicating whether there is production of item j in position n
h_j: Unit inventory holding cost for item j	δ_{ijn}: 1, If setup is incurred for the transition from the production of item i in position (n-1) to that of item j in the next position (n); 0, Otherwise
CO_t: Cost of overtime in period t	I_{jt}: Inventory of item j at the end of period t
CP_j: Unit cost of production for item j	O_t: Amount of overtime used in period t
d_{jt}: Demand of item j in period t	
C_t: Capacity in period t	
P_j: Unit processing time of item j	
N_t: Number of positions in period t	
$F_t(L_t)$: First (last) position in period t	
M_j: Minimum batch size of item j	

(P) Minimize $$\sum_i\sum_j\sum_n \delta_{ijn}SC_{ij} + \sum_j\sum_t h_j I_{jt} + \sum_j\sum_n CP_j X_{jn} + \sum_t CO_t O_t \qquad (0)$$

Subject to $$I_{jt} = I_{j(t-1)} + \sum_{n=F_t}^{L_t} X_{jn} - d_{jt} \qquad \forall t, j \qquad (1)$$

$$P_j X_{jn} \le C_t W_{jn} \qquad \forall t, j, n = F_t..L_t \qquad (2)$$

$$\sum_j \sum_{n=F_t}^{L_t} P_j X_{jn} + \sum_i\sum_j\sum_{n=F_t}^{L_t} ST_{ij}\delta_{ijn} \le C_t + O_t \qquad \forall t \qquad (3)$$

$$\sum_j W_{jn} = 1 \qquad \forall n \qquad (4)$$

$$\delta_{ijn} \ge W_{i(n-1)} + W_{jn} - 1 \qquad \forall i, j, n \qquad (5)$$

$$X_{jn} \ge M_j (W_{jn} - W_{j(n-1)}) \qquad \forall j, n \qquad (6)$$

W_{jn} binary $\forall j, n$, others positive (7)

In P, the objective function (0) minimizes the setup, inventory holding and production costs. Constraint (1) ensures that demand in a period is satisfied with no backlogging. Constraint (2) establishes the link between production and setup variables. Constraint (3) expresses the capacity limitations in period t and ensures that the capacity consumed in a period for production and setups does not exceed the available regular capacity and the overtime. Constraint (4) is introduced to prevent the model from assigning more than one item to a position and to enable the preservation of the setup state over idle periods. Constraint (5) forces the sequence dependent setup variable to take the value of 1 for the transition between the production of items in consecutive periods. (Note that the setup costs associated with the transition from an item to itself are defined to be 0). Constraint (6) imposes a minimum batch size restriction upon the production quantity of an item. In the case of setup costs and times satisfying the triangle inequality, constraint (6) can be dropped from the model. Otherwise, it has to be maintained in order to prevent the model from resorting to a change of setup state without an actual production change.

Even the feasibility problem is difficult for GLSP. For this reason it is important to develop heuristic approaches for finding good solutions.

3. A Mathematical Programming Based Two-Step Heuristic (TSH)

The solution of the integrated lot sizing and scheduling model can be approximated by a two-step heuristic (TSH) which decomposes the problem into lot sizing (LSM) and sequencing (SM) steps and solves them in an iterative manner. The two models can be presented as follows:

(LSM) min $\sum_{i,t} h_i I_{it} + \sum_t CO_t O_t$

such that $I_{i(t-1)} + X_{it} - I_{it} = d_{it}$ $\quad i = 1,\ldots,n,\ t = 1,\ldots,T$ (8)

$P_i X_{it} \leq C_t Y_{it}$ $\quad i = 1,\ldots,n,\ t = 1,\ldots,T$ (9)

$\sum_i P_i X_{it} \leq C_t + O_t - g(Y_t)$ $\quad t = 1,\ldots,T$ (10)

$X_{it} \geq M_{it} Y_{it}$ $\quad i = 1,\ldots,n,\ t = 1,\ldots,T$ (11)

$Y_{it} = 0$ or 1 $\quad i = 1,\ldots,n,\ t = 1,\ldots,T$ (12)

where $f(Y_t) = \min \sum_{ij} SC_{ij} Z_{ij}$

(SM_t) such that $\sum_{j=0, j\neq i}^{n} Z_{ji} = Y_{it}$, (except 0) $\quad i = 0,\ldots,n$ (13)

$\sum_{j=0, j\neq i}^{n} Z_{ij} = Y_{it}$, (except 0) $\quad i = 0,\ldots,n$ (14)

$\sum_{ij \in SxS} Z_{ij} \leq |S| - 1,$ $\quad S \subseteq N(Y_t), 2 \leq |S| \leq n$ (15)

$Z_{ij} = 0$ or 1 $\quad i = 0,\ldots,n,\ j = 0,\ldots,n$ (16)

where $g(Y_t) = \sum_{i,j} ST_{ij} Z_{ijt} + ST$ *(last item$_{(t-1)}$, first item$_t$)*

The LSM solves the lotsizing problem without considering setups. Then, corresponding Y variables are used as inputs in SM, which is solved for each period to determine the optimal TSP tour among the items selected by LSM. Then, the setup information obtained by the SM solution for each period is fed back to the LSM. The cost of the solution at each iteration is equal to the objective function of the LSM and SM added together. In the subsequent steps, the setup time required by the lotsizing solution is deducted from the available capacities in the LSM and the model is resolved. In these runs, the LSM faces reduced capacity and it may be obliged to shift the production of some items to other periods with more inventory in order to offset the effect of setups. Note that with the overtime option, we avoid the possibility of hitting an infeasible solution for the LSM in any step. The algorithm is designed to stop with a feasible solution whenever the total capacity required in a solution does not exceed the available capacity determined by the LSM (together with overtime). If the algorithm does not stop after a certain number of iterations, a feasible solution can be obtained by charging extra overtime penalty for each period with excess capacity requirements.

In this way, the model has been decomposed on a mathematical basis and the links between the two submodels have been developed iteratively. The preliminary experimentation results show that the TSH finds good feasible solutions within reasonable times but these are not given here due to space limitations.

4. Conclusions

In this paper, we have presented a mathematical model formulation for the integrated multi-item, capacitated lotsizing and sequencing problem with sequence-dependent setups, and proposed an iterative procedure for the solution. For the continuation of this study, we will conduct a detailed experimentation study to test the effectiveness of our solution procedure. We also consider enhancements to the problem such as the incorporation of multi-stages comprising a bottleneck stage followed by another stage such as containerization to come up with more realistic representations of process industry environments.

References

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