SOLVING THE P-CENTER PROBLEM WITH SIMULATED ANNEALING

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Abstract - *The p-center problem is a well-known NP-hard discrete location problem. This paper presents a metaheuristic, simulated annealing, for vertex p-center problem. Firstly, generic and problem specific decisions, and parameter setting of the algorithm are delineated. Secondly, the heuristic is tested on well-known p-center problems taken from literature. Thirdly, the results of the heuristic are compared with those of local search heuristics and other metaheuristic applications from the literature on the same test problem set. Simulated Annealing outperforms almost all local search heuristics in all problem instances. Moreover, it finds objective values as good as other metaheuristics in some problem instances. Finally, extension of the algorithm for the 'capacitated' case of vertex p-center is discussed.*

Keywords: *Modern Heuristics, Simulated Annealing, p-Center, Facility Location*

1. Introduction

The *p*-center problem is a well-known NP-hard discrete location problem. It consists of locating *p* facilities and assigning clients to them in order to minimize the maximum distance between a client and the facility to which he or she is assigned (i.e. the closest facility). This model is used, for example, in locating fire stations or ambulances, where the distance from the facilities to their farthest assigned potential client should be minimum. In Figure 1.1, an instance for 18 clients (n=18) and 3 facilities (p=3) can be seen. In this project, a metaheuristic, simulated annealing, is applied to vertex p-center problem (vertex means possible locations of the facilities are the clients' locations).

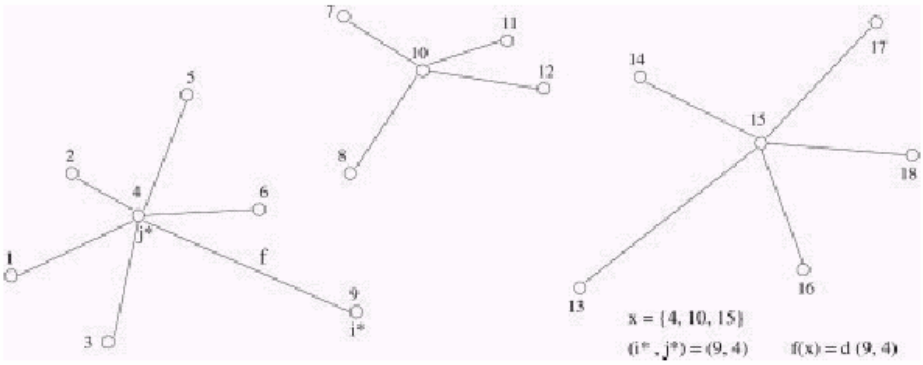


Figure 1.1. An instance for vertex 3-center problem with 18 clients (Source: Mladenovic et al. (2003))

Solution for vertex p-center can be represented with the facility node locations. In Figure 1.1 the solution is X = {4, 10, 15}. i* is the client node and j* is the facility location (and also client location) which give the objective function value, i.e. $d_{i^*j^*} = z=f(x)$. In Figure 1.1, i*=client node 9, j*=facility at node 4 (and client 4) and objective value is f(x) = d (9,4).

The next section will outline the simulated annealing algorithm; Section 3 will present the computational results of the algorithm; the last section will discuss the extension of heuristic to capacitated case and will conclude.

2. Simulated annealing algorithm

2.1 Generic and Problem Specific Decisions

During this project, a solution of the problem is represented as an array. First p elements of this array show the location (vertex number) of the facilities (also it represents corresponding client locations), other n-p elements of the array shows the non-facility client locations of the solution. With this data structure all possible, C(n,p), solutions to problem can be represented. This array representation can be seen in Figure 2.1.1.

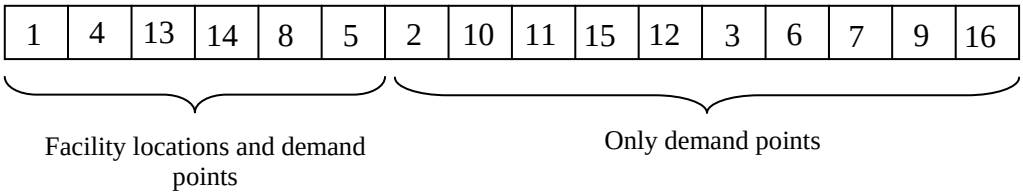


Figure 2.1.1. Array of a solution of X={1,4,5,8,13,14} for n=16 and p=6 problem

Neighborhood structure: Two neighborhood structures are observed in the algorithm. First results from simple move that randomly selects a facility node and a non-facility node, and change them, i.e. change a non-facility demand node as a facility demand node and change a facility node as a non-facility node. Second results from complex move that selects the facility node, which gives objective function value, and changes it by a randomly selected non-facility node if objective value is not changed for (1/4.number of facilities) iterations. Otherwise, complex move is the same as simple move. Simple move can be seen in Figure 2.1.2.

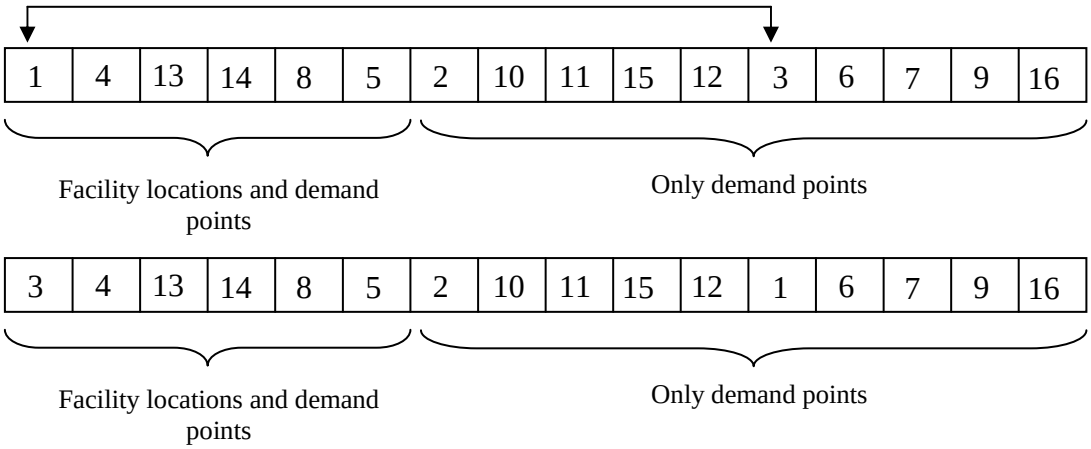


Figure 2.1.2. A simple move and new solution after move

Cooling Schedule: By the recommendation of Lundy and Mees (1986) and Reeves (1995) usage of geometric cooling schedule is decided. Where t represents temperature, that schedule is a(t)=t/(1+bt) where b is small, implying very slow cooling. At each temperature, only one iteration is done.

Stopping Condition: When t decreases to final temperature (t_f) or when the best known is found algorithm stops.

Starting Solution: Since performance of any metaheuristic should not depend on initial solution, any specific construction heuristic is not applied. Random construction of initial solution is done.

Steps of the Simulated Annealing algorithm after these decisions can be seen below:

Step 1. Set temperature (t) to starting temperature (t_s). Randomly construct an initial solution and find the objective that corresponds to this solution, assign it to current solution objective value (fcur).

Step 3. Repeat 3.1 Try a move (simple move for small problem, complex move for bigger problem)

3.2 Find objective corresponding to solution after move (fstr), find delta as delta=fstr-fcur

3.3 If (delta<0) then update current solution by making move and equating fcur to fstr

Else generate u $\in$ Uniform [0,1], if u < exp(-delta/t) update current solution by making move and equating fcur to fstr (i.e. acceptance probability of non-improving move is p_a=exp(-delta/t))

3.4 Decrease Temperature with geometric cooling schedule

Until (Temperature (t)$\leq$ Final Temperature (t_f)) or (best solution of algorithm = best known)

Step 4. Record the best solution and the iteration at which it is found, and finalize the program.

2.2 Parameter Setting of the Algorithm

For the algorithm described above the main parameters to be set is, number of iterations, starting temperature, final temperature and type of move. Firstly, the number of iterations directly affects the average running time of the algorithm. Moreover, it is expected that quality of the solution be affected by the number of iterations. Therefore the total number of iteration is taken as an important control parameter. Secondly, the quality of the solution depends on t_s and t_f, although the running time of the algorithm does not depend on t_s and t_f since geometric cooling schedule is used and the number of iterations is pre-determined. Thirdly, the type of move is expected to affect the quality of solution. Furthermore, since complex move requires more computation, average running time is influenced by the move type.

In order to understand the solution structure of the problem set (first 20 problems of p median problem set of Beasley (1990) and of web site of Beasley (1990)), random searches (always move new solution regardless of the objective value improvement in each iteration) for each problem instance are done. With some selected problem instances for experimentation (problem instances with bold numbers in Table 3.2), full factorial design of experiment is used in order to set the values for control parameters.

Experimentation values for number of iterations: Required number of iteration is taken as a function of number of clients (tnode) and number of facilities (tfac), i.e. iteration=f(tnode,tfac). Iteration numbers of best objective value found in the random searches of experimentation problems are regressed on tnode and tfac with many functional forms. Eventually, the form, iteration = e^a.tnode^b.tfac^c with a=6.75, b=0.5 and c=0.75, produces better results (Table 2.2.1). Experimentation values of the iteration are determined as the values come from the regression results r (i.e. r = $e^{6.75}$.tnode$^{0.5}$.tfac$^{0.75}$), r/2 and 2.r.

Table 2.2.1 Regression of ln(iteration of optimum values of randomsearch) on ln(tode) and ln(tfac) on 15 replications of experimentation problems

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.968617
R Square	0.938219
Adjusted R Square	0.927923
Standard Error	0.266791
Observations	15

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	12.97111	6.48555	91.1178	5.56E-08
Residual	12	0.854131	0.07117		
Total	14	13.82524			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	6.752551	0.653485	7.27262	9.84E-06	3.32873	6.176372	3.32873	6.176372
X Variable 1	0.503796	0.134917	3.76502	0.002696	0.21400	0.801924	0.214006	0.801924
X Variable 2	0.747353	0.07332	10.1439	3.07E-07	0.58400	0.903503	0.584002	0.903503

Number of iterations based on this regression (r) for each problem instance can be seen can be seen in table 2.2.2

Table 2.2.2 Required number of replications based on the regression

Pr	tnode	tfac	Iteration (r)	$e^{(6.75)}$	tnode$^{(0.50)}$	tfac$^{(0.75)}$
1	100	5	28557.17587	854.05876	10	3.343702

2	100	10	48027.25363	854.05876	10	5.623413
3	100	10	48027.25363	854.05876	10	5.623413
4	100	20	80771.89083	854.05876	10	9.457416
5	100	33	117590.7628	854.05876	10	13.76846
6	200	5	40385.94541	854.05876	14.14214	3.343702
7	200	10	67920.79345	854.05876	14.14214	5.623413
8	200	20	114228.7035	854.05876	14.14214	9.457416
9	200	40	192109.0145	854.05876	14.14214	15.90541
10	200	67	282851.7387	854.05876	14.14214	23.41834
11	300	5	49462.47952	854.05876	17.32051	3.343702
12	300	10	83185.64344	854.05876	17.32051	5.623413
13	300	30	189622.2613	854.05876	17.32051	12.81861
14	300	60	318905.3595	854.05876	17.32051	21.55825
15	300	100	467787.2497	854.05876	17.32051	31.62278
16	400	5	57114.35173	854.05876	20	3.343702
17	400	10	96054.50726	854.05876	20	5.623413
18	400	40	271683.1738	854.05876	20	15.90541
19	400	80	456914.8139	854.05876	20	26.74961
20	400	133	668969.7489	854.05876	20	39.16415

Experimentation values for t_s and t_f: From the random searches, two values are calculated for each problem instance. One is $delta_{max}$ which is the maximum delta (fstar-fcur) occurs during the random search of that problem, the other is $delta_{ave}$ which is the average of positive delta values in random search of that instance. Starting temperature should be hot enough to allow free exchange of neighbor solutions and to make the final solution independent of the starting solution. In other words, p_a should be high enough (like 0.99 or 0.95) to accept the move even in the case of worst objective deterioration. Hence $t_s(p_a)= [-delta_{max} / \ln(p_a)]$ for p_a values of 0.99 and 0.95 are taken as experimentation values. Through the end of the algorithm, temperature should be low enough to fully exploit neighborhood of the latter solutions. Therefore, $t_f(p_a)=[-delta_{ave}/\ln(p_a)]$ for p_a values of 0.01 and 0.05 are taken as control parameters.

Experimentation for move types: It can be expected that when the facility number increases the requirement for complex move increases. Therefore, the problem instances are divided into two groups; the small ones with tfac ≤ 10 and bigger ones with tfac > 10. Factorial design experiments are done separately for these two groups.

Results of the experimentation: Full factorial design of MINITAB®13 with 4 factors is used for experimentation. General experimentation frame can be seen in Table 2.2.3. Percentage gap from best known is used as a response for this experiment.

Table 2.2.3. Experimentation frame.

		Move								
		Simple move			Complex move - after (1/3*tfac) iteration			Complex move – after (1/4*tfac) iteration		
		Iteration			**Iteration**			**Iteration**		
		r/2	r	2*r	r/2	r	2*r	r/2	r	2*r
t_s	**t_f**									
t_s (0.99)	t_s (0.01)									
	t_s (0.05)									
t_s (0.95)	t_s (0.01)									
	t_s (0.05)									

For small problem case, 2 replications from Pr1, Pr7 and Pr11 of Table 3.1 are done for each combination. Therefore the experimentation is done for 144 replications. The results can be seen in table below.

Table 2.2.4. Experimentation results for the tfac<=10 experimentation problems. Significant factors are in bold characters

General Linear Model: C8 versus move, tstr, tfin, iteration

Factor	Type	Levels	Values
move	fixed	3	1 2 3
tstr	fixed	2	0.95 0.99
tfin	fixed	2	0.01 0.05
iteratio	fixed	3	r/2 r 2*r

Analysis of Variance for C8, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
move	2	0.0000016	0.0000016	0.0000016	0.16	0.688
tstr	1	0.0000047	0.0000047	0.0000047	0.46	0.497
tfin	**1**	**0.0003872**	**0.0003872**	**0.0003872**	**38.39**	**0.000**
iteratio	**2**	**0.0004489**	**0.0004489**	**0.0002244**	**22.25**	**0.000**
move*tstr	1	0.0000016	0.0000016	0.0000016	0.16	0.687
move*tfin	1	0.0000038	0.0000038	0.0000038	0.37	0.542
move*iteratio	2	0.0000066	0.0000066	0.0000033	0.33	0.722
tstr*tfin	1	0.0000057	0.0000057	0.0000057	0.56	0.455
tstr*iteratio	2	0.0000130	0.0000130	0.0000065	0.65	0.526
tfin*iteratio	**2**	**0.0003557**	**0.0003557**	**0.0001779**	**17.64**	**0.000**
move*tstr*tfin	1	0.0000038	0.0000038	0.0000038	0.37	0.542
move*tstr*iteratio	2	0.0000009	0.0000009	0.0000005	0.05	0.955
move*tfin*iteratio	2	0.0000057	0.0000057	0.0000028	0.28	0.756
tstr*tfin*iteratio	2	0.0000151	0.0000151	0.0000075	0.75	0.476
move*tstr*tfin*iteratio	2	0.0000016	0.0000016	0.0000008	0.08	0.926
Error	120	0.0012103	0.0012103	0.0000101		
Total	143	0.0024661				

Hence, for small problems, t_f and iteration are observed as significant factors. The response graphs for significant factors are in Figure 2.2.1.

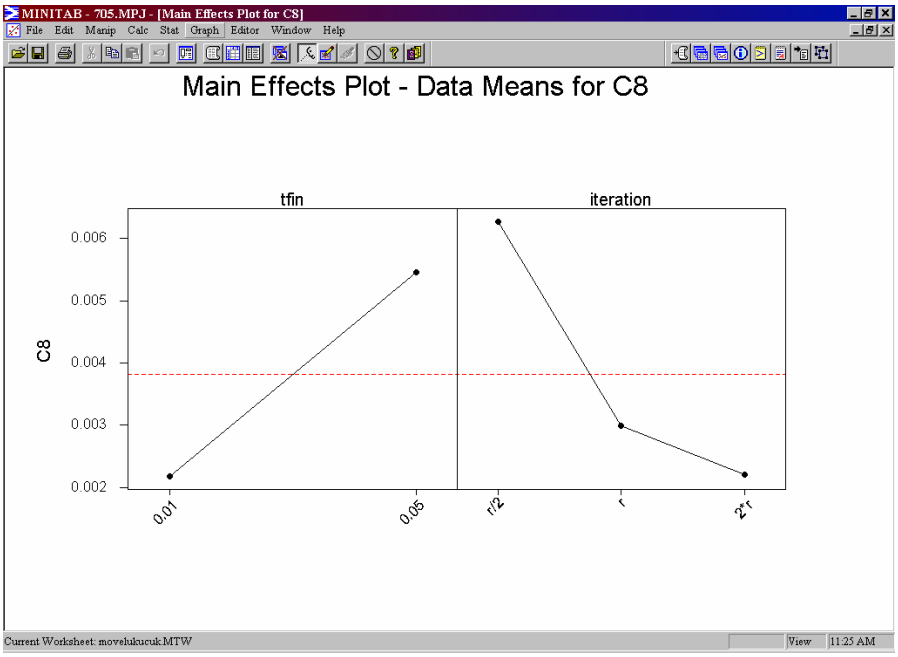


Figure 2.2.1. Main effects for significant factors of experimentation with tfac <=10

From the response graph of the experiment t_f (0.01) and iteration=r are selected although iteration (2.r) produces better solution quality. It is already expected that when the same algorithm runs for more step, it will produce better results. However, the improvement of the iteration (2.r) from the iteration (r) is very small (0.0002 percentage gap). On the other hand required amount of time for iteration (2.r) (5.052 seconds on average) is nearly two times higher than that for iteration (r) (2.583 seconds on average). Therefore, respecting to time efficiency trade off usage of r is decided. For insignificant factors, t_s=0.95 and simple move are chosen.

For bigger problems, 2 replications for Pr4, Pr8 and Pr19 of Table 3.1 are done for each combination. Therefore the experimentation is done for 216 replications. The results can be seen in table below

Table 2.2.5. Experimentation results for the tfac>10 experimentation problems.

General Linear Model: C8 versus move, tstr, tfin, iteration

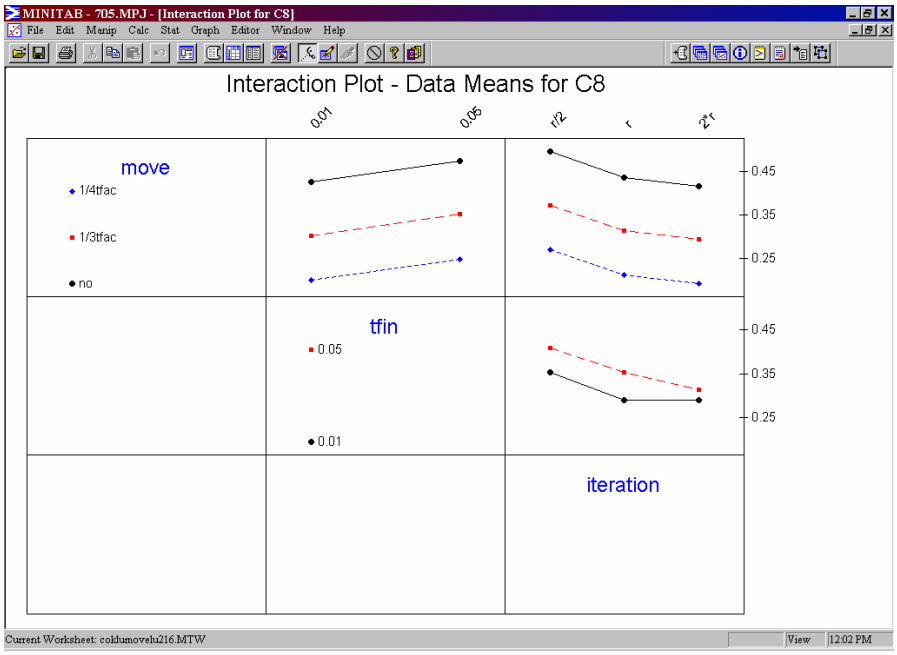
Factor	Type	Levels	Values
move	fixed	3	no 1/3tfac 1/4tfac
tstr	fixed	2	0.95 0.99
tfin	fixed	2	0.01 0.05
iteratio	fixed	3	r/2 r 2*r

Analysis of Variance for C8, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
move	**2**	**1.8182**	**1.8182**	**0.9091**	**8.08**	**0.000**
tstr	1	0.0019	0.0019	0.0019	0.02	0.898
tfin	**1**	**0.1269**	**0.1269**	**0.1269**	**1.13**	**0.290**
iteratio	**2**	**0.2432**	**0.2432**	**0.1216**	**1.08**	**0.342**
move*tstr	2	0.0000	0.0000	0.0000	0.00	1.000
move*tfin	2	0.0000	0.0000	0.0000	0.00	1.000
move*iteratio	4	0.0000	0.0000	0.0000	0.00	1.000
tstr*tfin	1	0.0091	0.0091	0.0091	0.08	0.776
tstr*iteratio	2	0.0174	0.0174	0.0087	0.08	0.926
tfin*iteratio	2	0.0154	0.0154	0.0077	0.07	0.934
move*tstr*tfin	2	0.0000	0.0000	0.0000	0.00	1.000
move*tstr*iteratio	4	0.0000	0.0000	0.0000	0.00	1.000
move*tfin*iteratio	4	0.0000	0.0000	0.0000	0.00	1.000
tstr*tfin*iteratio	2	0.0077	0.0077	0.0039	0.03	0.966
move*tstr*tfin*iteratio	4	0.0000	0.0000	0.0000	0.00	1.000

Error	180	20.2522	20.2522	0.1125
Total	215	22.4920		

As can be seen move type definitely affects the solution quality for this problem. In this table the t_f and iteration seem to be not effect the solution quality of the problem. However, if the move type is discarded, these two factor effect the solution quality. One can see this in the figure below:



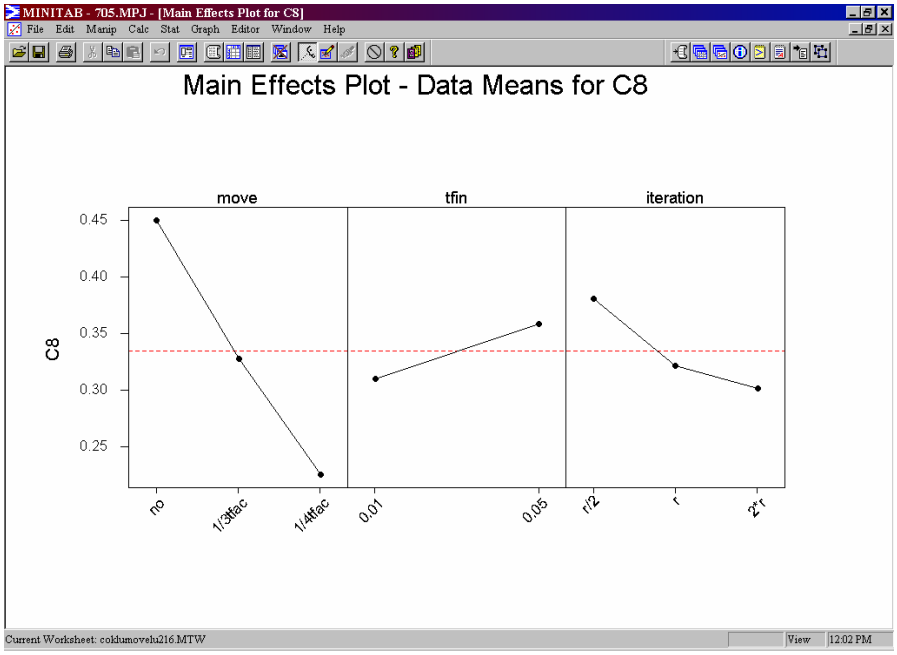


Figure 2.2.2 Main and interaction effect of the move, t_f and iteration for bigger problems

Therefore, move type, t_f and iteration are observed as significant factors. From the response graph of the experiment, complex move type, t_f (0.01) and iteration=r (although 2.r produce better results) are selected, and t_s is assigned to t_s (0.95).

3. Computational Results of the Algorithm

After setting the parameters of the algorithm, b parameter of the cooling schedule is determined as **b** = $[(t_s - t_f) / (\text{iteration}. t_s . t_f)]$ for each problem instance. In Table 3.1 all the parameters for each problem instance can be seen

Table 3.1. delta_{max} , delta_{ave}, tf(0.95), tf(0.01), iteration and b value for the 20 problem instances

Pr	tnode	tfac	Iteration	$e^{(6.75)}$	$\text{tnode}^{(0.50)}$	$\text{tfac}^{(0.75)}$	Delta_{max}	Delta_{ave}	ts(0.95)	tf(0.01)	b	Optimal	Fopt in random search
1	100	5	28557	854.0588	10	3.343702	66	2.871	1286.72	2.97	0.0000117631788	127	131
2	100	10	48027	854.0588	10	5.623413	65	10.740	1267.22	2.332	0.0000089115582	98	155
3	100	10	48027	854.0588	10	5.623413	125	28.015	2436.97	6.083	0.0000034141383	93	191
4	100	20	80772	854.0588	10	9.457416	79	13.940	1540.16	3.03	0.0000040779499	74	97
5	100	33	117591	854.0588	10	13.76846	56	12.270	1091.76	2.664	0.0000031839537	48	106
6	200	5	40386	854.0588	14.1421	3.343702	44	7.839	857.812	1.702	0.0000145165816	84	113
7	200	10	67921	854.0588	14.1421	5.623413	29	5.559	565.38	1.21	0.0000121417539	64	78
8	200	20	114229	854.0588	14.1421	9.457416	62	17.820	1208.73	3.87	0.0000022551227	55	101
9	200	40	192109	854.0588	14.1421	15.90541	35	3.563	682.35	0.774	0.0000067203115	37	98
10	200	67	282852	854.0588	14.1421	23.41834	42	11.350	818.82	2.465	0.0000014301506	20	77
11	300	5	49462	854.0588	17.3205	3.343702	32	5.466	623.86	1.19	0.0000169569585	59	64
12	300	10	83186	854.0588	17.3205	5.623413	43	8.432	838.316	1.831	0.0000065511176	51	59
13	300	30	189622	854.0588	17.3205	12.81861	27	7.770	526.385	1.687	0.0000031155954	36	83
14	300	60	318905	854.0588	17.3205	21.55825	37	11.370	721.342	2.469	0.0000012657105	26	76
15	300	100	467787	854.0588	17.3205	31.62278	66	13.821	1286.72	3.001	0.0000007106302	18	59
16	400	5	57114	854.0588	20	3.343702	22	3.985	428.906	0.865	0.0000201929151	47	51
17	400	10	96055	854.0588	20	5.623413	23	4.085	448.402	0.887	0.0000117130300	39	55
18	400	40	271683	854.0588	20	15.90541	28	3.037	545.88	0.659	0.0000055745930	28	62
19	400	80	456915	854.0588	20	26.74961	14	2.871	272.94	0.62	0.0000035219681	19	32
20	400	133	668970	854.0588	20	39.16415	26	3.237	506.889	0.703	0.0000021237033	14	53

50 replications for each of 20 problems of Beasley (1990) are done on Pentium III 500 MHz with 256 MB RAM. The results of these runs are summarized in shaded columns of Table 3.2. In this table, best known value (for first 12 problems it is optimum, for others it is best value found so far) and objective values in best replications of other heuristics is taken from Mladenovic et al. (2003). Variable neighborhood search (VNS), tabu search 1 (TS1), tabu search 2 (TS2) and Simulated Annealing of this paper (SA) are metaheuristics; others are local search heuristics (detailed description of heuristics can be seen in Mladenovic et al. (2003)). As can be seen in Table 3.2, SA is as capable of finding good deviations from best known as other metaheuristics in six problem instances (1, 2, 6, 7, 11, 16). Moreover, in two problem instances (18, 19) it does not have the worst deviation. SA dominates all the local search heuristics except multi interchange (MI) and multi interchange plus alternate (MA+I) in terms of objective value in best replication. MI and MA+I produce better results in problem instances 3, 4, 5, 10 and 12. However, on average SA (15.3 % deviation) produces better results than MI (18.2 % deviation). Since the other heuristics are solved on another machine, time comparison cannot be provided here. However, from Mladenovic et al. (2003) it is observed that SA average time (16.19) is not worse than any other metaheuristic (shortest average time is 16.78) although they are solved on a more sophisticated machine (Sun Sparc Station 10).

Table 3.2. Comparison of heuristics

				Objective value in best replication														% Deviation of best value from best known					Ave. Time (sec.)
Pr	# of node	# of fac.	Best Known	B-S	I	Gr	Gr +I	GrP	GrP +I	A	A+I	MI	MA	MA +I	VNS	TS1	TS2	SA	MI	VNS	TS1	TS2	SA	SA
1	100	5	127	184	133	166	133	148	133	148	148	127	127	127	127	127	127	127	0.0	0.0	0.0	0.0	0.0	0.73
2	100	10	98	142	102	155	109	131	119	112	112	98	98	98	98	98	98	98	0.0	0.0	0.0	0.0	0.0	1.12
3	100	10	93	137	108	191	102	168	102	131	99	93	95	93	93	93	93	95	0.0	0.0	0.0	0.0	2.2	1.11
4	100	20	74	109	135	157	111	127	111	111	111	74	79	74	74	74	74	79	0.0	0.0	0.0	0.0	6.8	3.72
5	100	33	48	76	74	143	90	106	70	87	87	48	59	48	48	48	48	51	0.0	0.0	0.0	0.0	6.3	5.35
6	200	5	84	113	93	115	100	99	94	100	92	84	84	84	84	84	84	84	0.0	0.0	0.0	0.0	0.0	2.38
7	200	10	64	87	77	97	84	85	84	77	77	64	66	64	64	64	64	64	0.0	0.0	0.0	0.0	0.0	3.54
8	200	20	55	76	92	110	84	94	94	86	86	58	67	57	55	55	55	58	5.5	0.0	0.0	0.0	5.5	7.92
9	200	40	37	59	57	98	68	76	73	71	71	46	56	42	37	37	37	42	24.3	0.0	0.0	0.0	13.5	11.65
10	200	67	20	33	53	70	70	70	70	77	42	30	34	29	20	20	20	33	50.0	0.0	0.0	0.0	65.0	24.78
11	300	5	59	71	68	71	66	98	68	64	64	59	59	59	59	59	59	59	0.0	0.0	0.0	0.0	0.0	2.26
12	300	10	51	78	62	83	69	77	77	77	59	51	72	51	51	51	51	52	0.0	0.0	0.0	0.0	2.0	3.59
13	300	30	36	58	54	72	59	58	55	59	54	41	41	39	36	36	36	39	13.9	0.0	0.0	0.0	8.3	10.81
14	300	60	26	43	60	76	76	60	60	76	76	36	40	35	26	26	26	29	38.5	0.0	0.0	0.0	11.5	31.35
15	300	100	18	30	40	59	51	49	49	44	44	29	35	27	18	25	18	27	61.1	0.0	38.9	0.0	50.0	48.6
16	400	5	47	55	59	61	47	54	47	52	48	47	47	47	47	47	47	47	0.0	0.0	0.0	0.0	0.0	2.11
17	400	10	39	55	53	53	50	51	51	49	43	40	40	39	39	39	39	40	2.6	0.0	0.0	0.0	2.6	3.97
18	400	40	28	44	50	62	50	61	50	50	50	38	40	34	28	28	35	35	35.7	0.0	0.0	25.0	25.0	19.95
19	400	80	19	31	36	41	41	38	38	36	36	28	28	26	19	23	19	23	47.4	0.0	21.1	0.0	21.1	49.17
20	400	133	14	22	37	49	49	34	32	44	44	26	29	22	14	22	14	26	85.7	0.0	57.1	0.0	85.7	89.67
																	Average:		18.2	0.0	5.9	1.3	15.3	16.19

Main shortcoming of the SA algorithm is that it cannot produce better results when the facility number is high (Table 3.3). Making more iterations for problem instances with facility numbers higher than 10 or applying more complex move like interchange (I) move of Mladenovic et al. (2003) can solve this problem.

Table 3.3. Performance of the SA algorithm.

			Best value	**# of times best found in 50 iteration**	**Average %deviation in 50 replication**
Pr	**tnode**	**tfac**	**SA**	**SA**	**SA**
1	100	5	127	49	0.001%
2	100	10	98	3	2.170%
3	100	10	95	4	5.230%
4	100	20	79	3	9.370%
5	100	33	51	5	12.590%
6	200	5	84	50	0.000%
7	200	10	64	2	3.312%
8	200	20	58	3	10.120%
9	200	40	42	1	21.380%
10	200	67	33	1	83.350%
11	300	5	59	7	12.220%
12	300	10	52	2	11.730%
13	300	30	39	2	19.680%
14	300	60	29	1	42.970%
15	300	100	29	2	104.970%
16	400	5	47	25	1.190%
17	400	10	40	1	6.560%
18	400	40	35	2	49.670%
19	400	80	23	4	97.840%
20	400	133	26	2	128.390%

4. Extension of the Algorithm for the 'Capacitated' Case and Conclusion

In the capacitated case of vertex p-center problem, main problem is the capacity constraint. When the algorithm is constructed as not to violate this constraint, it is seen that feasible move number from current solution decreases substantially, which causes cycles in the moves (same solutions in many steps). When infeasible moves are allowed, repaired infeasible moves cannot produce better results than even the most basic heuristics. Therefore, more complex move structures, which can foresee the future of the algorithm, will be used for the capacitated case. Namely, customer improvement graphs of Pallottino et al. (2003) will be used for this complex moves. However, the application is beyond the scope of this article and will be subject of another article.

In this paper, vertex p-Center problem was solved with Simulated Annealing algorithm. It produced better objective values than almost all local search heuristics, and produced objective values as good as other metaheuristics in some problem instances. However, algorithm should be redesigned in order to produce good results for problems with higher facility numbers. Finally, extension of the algorithm for capacitated case is discussed

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