

## ÇOK ÜRÜNLÜ TÜMLEŐİK YER BELİRLEME/ENVANTER PROBLEMİ

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**Özet:** Bu çalışmada, tek tedarikçinin birden fazla ürünü dağıtım merkezleri yoluyla perakendecilere gönderdiđi üç seviyeli bir dağıtım sisteminin tasarımı ele alınmıştır. Perakendecilerde belirsiz talep gözlenmektedir. Problem; tesis, taşıma, emniyet stoku ve ortak sipariř ile ortalama envanter maliyetlerinin toplamını minimize edecek ve müşteri hizmet gereklerini karşılayacak şekilde dağıtım merkezlerinin sayısının ve yerlerinin belirlenmesi ile perakendecilerin dağıtım merkezlerine atanmasıdır. Tümleriřik yer belirleme/envanter modeli; stratejik yer belirleme/tahsis etme kararlarına, risk ortaklaşmasının getireceđi faydaları ve ürünlerin gruplar halinde sipariř edilmesinden dođan kazançları dikkate alarak envanter yönetimi kararlarını da dahil etmektedir. Geliřtirilen dođrusal-olmayan tamsayı programlama modelini tümleriřik olarak çözebilmek için iki sezgisel yöntem geliştirilmiştir: (1) Geliřtirici tipte sezgisel yöntem, (2) Yapıcı tipte sezgisel yöntem. Her iki sezgisel yöntem de çok makul zamanlarda çözüm üretebilmektedir. Algoritmalar, oluşturulan problem örnekleri üzerinde test edilmiş ve çözüm kalitesi ile çözüm zamanı bazında karşılaştırılmıştır. Yapıcı tip sezgisel yöntem ile rasgele kümeleme ile bařlatılan geliřtirici tip sezgisel yöntemin çođu problem örneğinde daha iyi sonuç verdikleri bulunmuştur.

**Anahtar Kelimeler:** *Dağıtım Sistemi Tasarımı, Lojistik ve Envanter Yönetimi, Matematiksel Programlama, Ortak Sipariř Politikaları, Tesis Planlama ve Tasarımı*

## MULTI-ITEM INTEGRATED LOCATION INVENTORY PROBLEM

**Abstract:** In this study, the design of a three-level distribution system is considered in which a single supplier ships several items to the retailers via a set of distribution centers (DC), and stochastic demand is observed at the retailers. The problem is to specify the number and location of the DCs, and the assignment of the retailers to the DCs in such a way that the sum of facility, transportation, safety stock, and joint ordering and average inventory costs is minimized, while customer service requirements are satisfied. Single source constraints are imposed on the assignment of the retailers to the DCs. The integrated location/inventory model incorporates the inventory management decisions into the strategic location/allocation decisions by considering the benefits of risk pooling and the savings that result from the joint replenishment of a group of items. We formulate a non-linear integer-programming model and develop two heuristic methods to solve it: (1) Improvement type heuristic, and (2) Constructive type heuristic. Both heuristics are able to generate solutions in very reasonable times. The algorithms are tested on the generated problem instances and compared in terms of their solution quality and computation time. It is found out that the constructive type heuristic and improvement type heuristic initiated with randomized clusters provide better solutions in most of the problem instances.

**Keywords:** *Distribution System Design, Logistics and Inventory Management, Mathematical Programming, Joint Replenishment Policy, Facility Planning and Design*

### 1. Introduction

The distribution/location family of problems covers formulations, which range in complexity from simple single-item linear deterministic models to multi-item nonlinear stochastic versions. Although the range of previous work on distribution/location problems is quite extensive, comparatively less attention has been paid to warehouse location problems in which inventory costs are a significant determinant of the warehouse locations. However, the inventory decisions have a significant impact on service levels and total cost in the management of the supply chain. Moreover, the cost and service level performance of a distribution system depends heavily on the interaction between the physical design of the distribution system and the inventory control system. Most of the formulations so far focus on mixed integer programming representations, in which the demands at a set of specified locations are assumed fixed and known with certainty, and inventory costs are either neglected or assumed to be unrelated to the DC location decisions. The models generally include integer variables for locating plants and/or DCs in given locations and allocating customers to the DCs, and continuous variables for determining flows of products through the system. Geoffrion and Graves (1974), Pooley (1994), Hindi, Basta and Pienkosz (1998), Tragantalerngsak, Holt and Rönnqvist (2000), and Melkote and Daskin (2001) are some studies

that do not consider the inventory costs explicitly in their distribution/location models, while Barahona and Jensen (1998), Sery, Presti and Shobry (2001), Karabakal, Günel and Ritchie (2000), Robinson, Gao and Muggenborg (1993) address the distribution design problem in which they charge inventory costs for the material flow through the DCs. These approaches fail to recognize the interdependence between location and inventory decisions and misrepresent inventory costs in the location models (Daskin, 1985). However, there are some recent studies, Nozick and Turnquist (1998), Nozick and Turnquist (2001), Erlebacher and Meller (2000), Daskin, Coullard and Shen (2003), and Daskin, Coullard and Shen (2002), which consider the inventory policies explicitly in their integrated location/inventory models.

In this study, we propose a modeling and solution approach for the integrated inventory/location problems in a three-echelon distribution system. In this system, multiple items are delivered from a single supplier to the retailers through the DCs, where the items are controlled with a multi-item inventory policy. Our study differs from the earlier multi-item distribution system design problems in the sense that we consider joint replenishment policy at the DCs and formulate the problem in an integrated way, and propose a solution methodology for the resulting nonlinear integer programming model.

## **2. Analysis of the Integrated Location/Inventory Problem**

We consider the design of a distribution system, in which a single supplier ships several items to the retailers via a number of DCs to be established. The locations of the single supplier and the retailers are assumed known and fixed. Random demand is observed for all items at the retailers. Due to the variability in demand at the retailers, it is required that some amount of safety stock is held therein to achieve suitable service levels. However, to take the advantage of risk pooling, inventory is held only at the retailers that are set as DCs. Each DC can maintain and distribute all the items. All of the retailers are considered to be candidate locations for DCs. The inventory costs incurred by small amounts of inventory held at the retailers for short periods of time are ignored. Single source constraints are imposed on the assignment of the retailers to the DCs, and DCs are assumed to be uncapacitated. In this multi-item distribution system, the DCs replenish their inventories by ordering each item from the single supplier. A multi-item inventory system is assumed at each DC. By implementing a joint replenishment policy, each DC determines the items that should be procured in each order and the ordering interval for each item.

Given this background, we can state the problem as follows: Given a collection of retailers, each with uncertain demand for several items, determine how many DCs to locate, where to locate and which retailers to assign to each DC, such that the sum of fixed facility location costs, inbound and outbound transportation costs, joint ordering and average inventory costs, and safety stock costs is minimized, while ensuring a specified level of service at the retailers for each item.

We formulate a multi-item integrated location/inventory model that considers the facility costs, inbound and outbound transportation costs, safety stock costs and the cost implications of a joint replenishment policy used at the DCs. Due to the complexity of the exact analysis of the multi-item inventory problem, we approximate the joint replenishment policy by assuming deterministic demand. The model thus formulated turns out to be a nonlinear integer programming model.

## **3. Solution Approach and Computational Results**

In order to solve the integrated location/allocation model, we develop two heuristic procedures: *improvement type* and *constructive type* heuristics, which are based on some traditional heuristic algorithms developed for the location/allocation problems (Kuehn and Hamburger, 1963; Feldman, Lehrer and Ray, 1966; Kelly and Khumwala, 1974; Whitaker, 1985). Both heuristic approaches can handle the difficulties posed by the nonlinear terms in the objective function and especially by the structural properties of the joint replenishment policy.

Both algorithms are tested on the generated problem instances and compared in terms of their solution qualities and computation times. It is found out that it is difficult to favor one of the heuristics according to the quality of the solutions obtained. However, it is seen that the best objective function values are mostly obtained by the constructive type algorithm and/or the improvement type algorithm that is started using random clusters. The improvement type algorithm initiated by the  $p$ -median solution performs better than the other algorithms, only when the importance of the transportation costs is increased significantly. The solution times of the algorithms can be regarded as low and even negligible, when the strategic nature of the problem handled is considered. The computational results obtained by each heuristic are compared to the solution of the  $p$ -median problem as well. It is observed that we can obtain reduction in total cost and the number of DCs located when we solve the problem in an integrated way rather than solving it by  $p$ -median problem. We also perform sensitivity analysis to find out how the

solutions and performance of the solution approaches respond to the changes in the transportation, inventory, and joint ordering cost parameters, and also variability of the demand.

#### 4. Further Research Issues

Although we obtain solutions to the multi-item integrated inventory/location problem in reasonable times by the application of proposed heuristics, we still do not know how far the best solution generated is from the optimal solution. The results of the experiments suggest that the algorithms proposed provide improved solutions with respect to the  $p$ -median solution, and therefore they are useful particularly in the absence of any efficient exact optimization methods. In order to evaluate the effectiveness of the heuristic algorithms for the multi-item integrated location/inventory problem on a more sound basis, the development of a lower bound can be considered as a future research study.

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