SABİT İŞ ÇİZELGELEME PROBLEMİ: LİTERATÜR DEĞERLENDİRMESİ

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Özet: Sabit iş çizelgelemesi hizmet ve imalat sistemlerinde sıkça rastlanan bir problemdir. Problem, sabit geliş ve termin zamanları olan aktivitelerin paralel özdeş kaynaklara yerleştirilmesi olarak tanımlanabilir. Pratik önemine rağmen, literatürde sabit iş çizelgelemesi üzerine çok az sayıda çalışma bulunmaktadır. Bu çalışmalarda, kaynak kullanım maliyetlerinin minimizasyonu (taktiksel problem) veya aktivitelerin toplam kar maksimizasyonu (operasyonel problem) hedeflenmiştir. Taktiksel problem tüm işlerin işlenmesini, operasyonel problem ise işlenecek aktivite kümesinin belirlenmesini gerektirmektedir. Biz bu çalışmada, sabit iş çizelgelemesi problemi üzerine literatürde yer alan çalışmaları, çalışma zamanı (working time), yaygınlık zamanı (spread time) veya işleyebilirlik (eligibility) kısıtları altında sınıflandırdık ve karşılaştırmalı değerlendirmeler yaptık. Ayrıca gelecek çalışmalara ışık tutması amacıyla açık araştırma alanlarını belirledik.

***Anahtar Kelimeler*:** *Interval Scheduling, Sabit İş Çizelgelemesi*

THE FIXED JOB SCHEDULING PROBLEM: A LITERATURE REVIEW

Abstract: Fixed Job Scheduling is an important problem in production and service environments. The problem can be defined as scheduling jobs that have fixed ready times and deadlines to identical parallel machines. Despite its practical importance, there are very few studies in literature about this problem. In existing studies, the aim is to minimize the number of machines (tactical problem) or maximize the total profit of jobs (operational problem). In this study, we consider the problem under working time, spread time and eligibility constraints. We review the related literature and state open areas for research.

***Keywords*:** *Interval Scheduling, Fixed Job Scheduling*

1. Introduction

The IS problem is typical for reservation systems and has many real life applications. There are n independent jobs available to be processed on m parallel machines. The time window of job j is specified by a release (ready) time r_j and a due time (deadline) d_j. If the job cannot be delayed after its ready time, then the IS problem becomes a Fixed Job Scheduling (FJS) problem. In this study, we consider the FJS problem, which has the following two variants based on objective functions. The first variant is the Operational Fixed Job Scheduling (OFJS) problem where each job has a weight w_j, and maximizing weighted number of processed jobs with a given number of processors is of concern. The second variant is the Tactical Fixed Job Scheduling (TFJS) problem where there is a fixed cost c_k of each machine, and the objective is the minimization of the total cost of the machines needed to process all jobs.

There are many practical applications of TFJS and OFJS. Kroon (1990) uses the TFJS to model in tactical capacity planning of aircraft maintenance personnel. Kroon *et al.* (1995) addresses the OFJS variant of the same case. Another application of the TFJS problem is the Bus Driver Scheduling Problem (see Fischetti *et al.*, 1987). The OFJS problem may be observed in scheduling earth-observing satellites (Wolfe and Sorensen, 2000).

It is assumed that each job needs to be processed on at most one machine without preemption or splitting, and all machines are identical and eligible for processing all jobs at all times. In practice, additional constraints such as eligibility, working time, spread time, or availability, and parameters that are more general such as uniform machines are needed.

2. Basic Models of the FJS Problem

Bouzina and Emmons (1996) provide a polynomial time algorithm for solving the OFJS problem with the objective of the maximization of the number of processed jobs. They also formulate the total weight maximization problem as a Minimum Cost Network Flow problem. The TFJS problem is studied by Dantzig and Fulkerson (1954), Gertsbakh and Stern (1978), Hashimoto and Stevens (1971), and Gupta *et al.* (1979).

3. The FJS Problem with Working Time Constraints

The only study in literature regarding the TFJS problem with working time constraints is that of Fischetti, Martello and Toth's (1989). The authors deal with the Bus Driver Scheduling Problem where the objective is minimizing the number of drivers necessary to perform all duties, and a crew cannot work for more than a given working time T in a day. The authors prove that the problem is NP-hard in the strong sense. They present an $O(n^2)$ algorithm that optimally solves the preemptive version, and develop a branch and bound algorithm for the problem. In a later study, Fischetti *et al*. (1992) provide some polynomial time approximation algorithms for the problem. Bouzina and Emmons' (1996) deal with the preemptive OFJS problem maximizing the number of jobs processed with working time constraints, and provide an optimal polynomial time algorithm. They also show that the feasibility problem of OFJS where the number of jobs is maximized is NP-complete.

4. The FJS Problem with Spread Time Constraints

The only study in FJS literature with working time constraints is that of Fischetti, Martello and Toth's (1987). The authors deal with the Bus Driver Scheduling Problem with a constraint on spread time of the drivers. The authors prove that the problem is NP-hard in the strong sense. They provide a polynomial time algorithm that solves the preemptive version. A Branch and Bound algorithm is described for solving the problem. In a later study (Fischetti *et al*., 1992), some polynomial time approximation algorithms for the problem are provided.

5. The FJS Problem with Eligibility Constraints

When each machine is eligible to process only a subset of jobs in FJS, several job and machine classes may be formed based on similarities. For each combination of a job class and a machine class, the feasibility of assigning a job to a machine is established in advance by using zero/one matrices. The TFJS problem with identical machine classes and one job class is considered by Hashimoto and Stevens (1971), Gertsbakh and Stern (1978), and Gupta *et al*. (1979). Arkin and Silverberg (1987) show that the corresponding feasibility problem is NP-complete. Dondeti and Emmons (1992) prove that the TFJS problem with minimization of cost objective with special matrices can be solved in polynomial time. They also show that, for these special matrices, the tactical version of the problem that finds the required number of machines in each machine class can be solved in polynomial time. Kolen and Kroon (1992) study the TFJS problem that minimizes the total cost of machines. Dondeti and Emmons (1993) formulate the preemptive feasibility problem as a set of transportation problems. Kroon *et al.* (1997) present a complete classification for the TFJS problem with eligibility constraints in terms of complexity. Jansen (1994) provides an approximation algorithm for the TFJS problem with eligibility and availability. Santos and Zhong (2001) develop a genetic algorithm for the TFJS problem with eligibility constraints. Arkin and Silverberg (1987) show that the feasibility problem for the OFJS problem with eligibility is NP-complete. Kolen and Kroon (1991) study the OFJS problem with different eligibility structures. Kroon *et al.* (1995) provide a Lagrangean relaxation based approximation algorithm for the problem.

The Hierarchical Class Scheduling (HCS) problem is defined as a problem with a upper triangular (or lower triangular) square matrix, which implies that a machine in class f can process jobs in classes e such that $e \leq f$ (or $e \geq f$). Kroon *et al.* (1997) show that the feasibility problem for the tactical HCS problem is NP-complete when $F \geq 4$. Dondeti and Emmons (1992) show that the tactical HCS problem can be solved in polynomial time if $F \leq 2$. Dondeti and Emmons (1993) propose a greedy algorithm for the preemptive HCS problem with $F > 2$. For the operational version, Kolen and Kroon (1991) prove that the problem is NP-hard in the strong sense when $F > 1$. Bouzina and Emmons (1995) conjecture that the problem of maximizing the number of jobs is NP-hard. Dondeti and Emmons (1993) prove that the feasibility problem for the tactical One-or-all Class Scheduling problem, where there are E machine classes and E+1 job classes, is NP-complete.

6. Further Topics in FJS

When any of the machines is available only for a specified time interval, the resulting problem is FJS with availability constraints. The FJS problem with eligibility constraints, and the FJS problem with availability constraints are closely related. The operational problem with availability constraints seems also related with the OFJS problem with spread time constraints.

As opposed to the situation where all machines are identical in terms of processing speeds, the machine speed factors may determine the processing times of the jobs, in which case the problem is referred to as FJS problem with uniform machines. All the problems analyzed in the previous chapters for identical machines can be extended to the uniform parallel machine case.

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