MULTIFACILITY LOCATION ALLOCATION PROBLEM

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Abstract: In this paper, we study the capacitated squared Euclidean distance and general ℓ_p distance location-allocation problem in the continuous space which seeks the optimum locations of facilities and the allocation of their products to customers. We propose a mixed-integer programming formulation for the problem. The proposed formulation gives the optimum solution for the rectilinear distance version and approximate solutions for the general ℓ_p case for the original problem. While formulating the problem we use a cell based approach which uses a two-phase heuristic to discretise the continuous space. Computational results are provided for some test problems.

Keywords: *Mathematical Programming, Facility Location*

1. Introduction

Facility location problems are concerned with determining the best location for facilities so as to minimize the total cost (or maximizing the profit) while satisfying the demands of the existing facilities. When the problem involves more than one facility to be located, consequently, the allocation problem is born (i.e., which new facility will serve which existing facility). If the new facilities have predetermined fixed capacities and multi-sourcing is permitted (i.e., more than one new facility can respond to one existing facility's requests), the next question that needs to be answered is what amount of these requests will be covered by each new facility.

In that study, we propose some heuristics for the capacitated location-allocation problem where multi-sourcing is allowed. We try to find answers to the following questions when the planar coordinates of *m* existing facilities with fixed demands are given along with the number of new facilities to be located, their capacities and the transportation costs per unit distance:

- What will be the coordinates of the new facilities which minimize the total transportation cost from the existing facilities to the new facilities?
- Which new facility will serve which existing facilities in what quantity?

We assume that there is no interaction among the new facilities and also existing facilities cannot interact with each other (i.e. no existing facility can share its demand with other existing facilities).

Capacitated facility location-allocation problem (CFLAP) can be formulated as follows:

Minimize $$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} w_{ij} \ell_p(\mathbf{X}_i, \mathrm{P}_j) \quad (1.1)$$

Subject to $$\sum_{j=1}^{m} w_{ij} = s_i \quad \text{for } i=1,...,n \quad (1.2)$$

$$\sum_{i=1}^{n} w_{ij} = d_j \quad j=1,...,m \quad (1.3)$$

$$w_{ij} \geq 0;\ i=1,...,n;\ j=1,...,m$$

where,
m is the number of existing facilities,
n is the number of new facilities to be located,
w_{ij} is the quantity allocated from new facility *i* to existing facility *j*
d_j is the demand of existing facility *j*
s_i is the capacity of new facility i
c_{ij} is the cost of transporting one unit of allocation one unit of distance
$\mathbf{X}_i = (x_i, y_i)$ and $\mathbf{P}_j = (a_j, b_j)$ are the planar coordinates of the new facility *i* and the existing facility *j* respectively and $\ell_p(\mathbf{X}_i, \mathbf{P}_j)$ is the ℓ_p distance between $\mathbf{X}_i$ and $\mathbf{P}_j$ given by $[|x_i-a_j|^p+|y_i-b_j|^p]^{1/p}$, $p\geq 1$

We assume that that total supply equals total demand, however if this is not the case, dummy facilities can be added to the model. The metric used in (1.1) to measure the distance, takes special names for different values of *p*. For *p*=1, it is called *rectangular distance* or *rectilinear distance* and for *p*=2, the distance measure becomes *straight-line* (*Euclidean*) *distance*.

2. Previous Research

Literature review reveals that there are many researches conducted about different versions of facility location problems. While some of these studies focus on uncapacitated versions, (e.g. Salhi and Gamal, 2003; Lozano *et al.,* 1998), the others focus on the capacitated versions (e.g. Sherali *et al.,* 2002; Eben-Chaime *et al.,* 2002). It is also possible to make a classification as "pure location problems" and "location-allocation problems". Pure location problems, as given in Francis *et al.* (1992), seek to find only location of new facilities. These types of problems are principal subproblems of more general location-allocation problems. Depending on the distance measure used to weigh the flows among facilities, there exists different solution methods for pure location problems. Researches conducted on the CFLAP are limited in number. Sherali *et al.* (1994) try to solve the rectilinear distance CFLAP. They discretise the problem by using the fact that optimal locations of the new facilities lie in the convex hull of the grid points of vertical and horizontal lines drawn through the existing facility locations. The resulting problem is then linearized by using the reformulation and linearization technique (RLT). After the application of RLT, the problem turns into a linear mixed-integer programming problem. The authors propose Lagrangean dual solution of the problem using conjugate subgradient algorithm to get a lower bound. They also propose a heuristic for deriving upper bounds and next present a branch-and-bound algorithm to solve the linear mixed-integer problem. The authors are able to solve problems of sizes ranging from (n, m)=(4, 20) to (5, 12) within 1% optimality and problems of size (5, 20) within 5% optimality. For the squared Euclidean distance CFLAP, Sherali and Tuncbilek (1992) fix the w_{ij}'s in the objective function and reformulate the problem in the space of w variables resulting in a convex maximization problem. Next, they present upper and lower bounding functions for the problem and use a branch-and-bound enumeration algorithm. Their solution methods are able to solve problems of sizes ranging from (n, m) = (6,120) to (20, 60) within 2% of optimality. Sherali *et al.* (2002) extend the previous works by introducing global optimization procedures for Euclidean and ℓ_p distance version of the CFLAP. The authors design a branch-and-bound algorithm based on a partitioning of the allocation space.

All of the solution methods suggested for the CFLAP seem to be computationally intractable; they include somewhat complex procedures and they are unable to solve large sized problems. Therefore heuristics seem to be more suitable for CFLAP.

3. Discrete Formulation of the Problem

Assume that there exists a set of fixed candidate facility locations for the CFLAP which includes the optimum locations. In that case it is possible to formulate the CFLAP discretely as follows:
DCFLAP:

$$\text{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{K} \mathrm{C}_{ijk} w_{ijk} \tag{3.1}$$

$$\text{Subject to} \quad \sum_{k=1}^{K} \sum_{i=1}^{n} w_{ijk} = d_j \qquad j=1,...,\text{m} \tag{3.2}$$

$$\sum_{k=1}^{K} Y_{ik} = 1 \qquad i=1,...,\text{n} \tag{3.3}$$

$$\sum_{j=1}^{m} w_{ijk} = s_i Y_{ik} \qquad i=1,...,\text{n};\ k=1,...,\text{K} \tag{3.4}$$

$Y_{ik} \in \{0, 1\}$, $w_{ijk} \geq 0$; i=1,...,n; j=1,...,m, k=1,...,K where
K is the number of candidate locations,
w_{ijk} is the amount sent from new facility i located in candidate point k to existing facility j,
Y_{ik} is the binary variable taking value of one if the ith new facility is opened in candidate location k and zero otherwise,
C_{ijk} is a constant which is calculated by multiplying C_{ij} by the ℓ_p distance between candidate point k and existing facility j

Wendell and Hurter (1973) shows that optimal solution for the rectilinear distance location problem occurs at the intersection points of vertical and horizontal lines drawn from the existing facilities and inside the convex hull of existing facility locations. Then by using the intersection points as the candidate locations, rectilinear distance CFLAP can be formulated as DCFLAP. However for the squared Euclidean and general ℓ_p cases, it is more difficult to find candidate locations which contains optimum locations because of the fact that optimum locations can occur anywhere in the continuous plane. One way of overcoming this problem is to find a set of candidate points which may not contain the exact locations of the optimum solution but nearby optimum locations. This can be achieved by constructing a grid whose some of cells contain the optimum locations. Representing each cell by its centroid and taking these points as the candidate locations, general ℓ_p distance problems can be formulated as DCFLP and solved approximately. After solving DCFLAP, an improvement scheme can be applied to further strengthen the solution. We propose a two-phase improvement scheme. Note that CFLAP turns into a transportation problem for fixed values of $\mathbf{X}_i$ and it turns into a pure location problem which is separable over i for fixed values of w_{ij}. Given some initial locations, solving the transportation problem and next the n location problems and repeating those steps until no improvement is possible in the objective function value is the underlying logic of our proposed two-phase method.

In order to solve DCFLAP, another way is to use lagrangean relaxation and subgradient optimization. By relaxing constraints (3.2), the problem decomposes into n sub problems, each of which can be easily solved by a search procedure. For deriving an upper bound in each iteration of the subgradient optimization, solution of the transportation problem or for a tighter upper bound, the result of the two phase method can be used.

4. Computational Results

We have tested our proposed algorithm in several test problems compiled from the literature. While determining candidate locations for the DCFLAP formulation, we first ran the-two phase procedure 25 times with randomly generated integer initial locations. Using the results of the 25 runs, we constructed a 25-cell grid (5 by 5). Then we considered the cells which contain at least one result of the 25 two-phase runs as candidate locations and neglected the remaining ones. Among the tested problems, optimum solutions have been found for 14 problems whereas there was less than 2 percent optimality gap for the remaining 4 problems. Each of the test problems has been solved in less than one minute.

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