KARIŞIK MODELLİ ÜRETİM HATTI SIRALAMA PROBLEMİNE LAGRANGE GEVŞETME YAKLAŞIMI

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Özet: Bu çalışmada, değişik amaç fonksiyonları içeren karışık modelli üretim hattı sıralama problemi üzerinde durulmuştur. Karışık Modelli Üretim Hattı'nda (KMH) ürünler konveyör üzerinde taşınmakta, ve aynı ürünün değişik modelleri, karışık şekilde aynı hat üzerinde üretilmektedir. Toplam çevrim zamanının ve toplam akış zamanının enazlanması olmak üzere iki değişik amaç fonksiyonu üzerinde çalışılmıştır. Bu iki amaç fonksiyonunu için Lagrange gevşetmeye dayalı bir yaklaşıklama algoritması geliştirilmiştir. Algoritma çözüm kalitesi ve çözüm süresi ölçütleri kullanılarak test edilmiş ve büyük boyutlu problemler için optimale çok yakın sonuçlara kısa zamanlarda ulaşılmıştır.

Anahtar Kelimeler: *Karışık Modelli Üretim Hattı, Çizelgeleme, Lagrange Gevşetme*

A LAGRANGEAN RELAXATION APPROACH FOR THE MIXED MODEL FLOW LINE SEQUENCING PROBLEM

Abstract: In this study, a mixed model flow line sequencing problem with two objectives is considered. Mixed model flow line (MML) is known to be a special case of production lines where products are transported on a conveyor belt, and different models of the same product are inter-mixed to be assembled on the same line. We have focused on product-fixed, rate-synchronous lines with variable launching. Our objective functions are minimizing makespan, and minimizing total flow time. An approximation algorithm based on Lagrangean relaxation is developed especially for large-size instances of the problem with these objectives, and tested in terms of solution quality and computational efficiency.

Keywords: *Mixed Model Flow Lines, Scheduling, Lagrangean Relaxation*

1. Introduction

In a mixed model flow line, there are N jobs that should be processed by M stations in the same processing order. There are a total of R product models, and each job belongs to a particular model. The transfers between the stations are synchronous, i.e., the transfers take place at the times when all stations finish their jobs. Then, we may define two different objectives as follows: Objective makespan is $\sum_{j=1}^{N+M-1} C_j$. Total flow time (total throughput time) objective becomes $N\sum_{j=1}^{M} C_j + \sum_{j=M+1}^{M+N-1} (N+M-j)C_j$. In this study, we develop an approximation algorithm for the problem of minimizing makespan on synchronous mixed model lines based on Lagrangean relaxation. The only study on the problem with makespan objective is that of Soylu (2002), in which an optimizing Branch and Bound algorithm is developed for small instances.

2. Problem Definition

Each job belongs to one model, and D_r denotes the demand (or order) for model. Let p_{rm} denote the processing time of a job a job of model r on station m. A binary variable X_{rn} takes the value of one if a job of model r occupies the n^{th} position in the schedule, and zero otherwise. The problem with makespan objective can be modeled as a mixed integer program as follows:

$$z = \text{Min} \sum_{j=1}^{N+M-1} C_j \qquad (0)$$

subject to:

$$\sum_{r=1}^{R} X_{rn} = 1 \qquad n = 1,..., N \qquad (1)$$

$$\sum_{n=1}^{N} X_{rn} = D_r \qquad r = 1,..., R \qquad (2)$$

$$C_j \geq \sum_{r=1}^{R} p_{rk} X_{r\,(j-k+1)} \qquad j,k \in JK=\begin{Bmatrix} j=1,...,M-1 & k=1,...,j \\ j=M,...,N & k=1,...,M \\ j=N+1,...,N+M-1 & k=j-N+1,...,M \end{Bmatrix} \tag{3}$$

$$X_{r\,n} \in \{0,1\} \qquad r = 1,..., R \qquad n = 1,..., N \tag{4}$$

$$C_j \geq 0 \qquad j = 1,..., N+M-1 \tag{5}$$

The objective function expressed in (0) minimizes makespan. Constraint set (1) guarantees that every position in the sequence is occupied by one job. Constraint set (2) implies that the demand for each model is exactly satisfied. Constraint set (3) identifies each cycle time as the maximum of the processing times of all jobs being processed in that cycle. Finally, constraints (4) and (5) are the integrality and nonnegativity constraints, respectively. The problem with total flow time objective has the same set of constraints.

The problem with total flow time objective has the same set of constraints with the following objective function:

$$z = \text{Min} \qquad N\sum_{j=1}^{M} C_j + \sum_{j=M+1}^{M+N-1} (N+M-j)C_j \tag{0'}$$

3. Lagrangean Relaxation

We choose to relax constraint set (3) with Lagrange multipliers, λ_{jk}. For the makespan objective, the relaxed model can be expressed as follows:

$$LR(z) = \text{Min} \sum_{j=1}^{N+M-1} C_j - \sum_{j,k \in JK} \lambda_{jk} C_j + \sum_{j,k \in JK} \lambda_{jk} \sum_{r=1}^{R} p_{rk} X_{r(j-k+1)}$$

subject to: (1), (2), (4), (5), and $\lambda_{jk} \geq 0$, $j,k \in JK$.

When the objective is the minimization of total flow time, the relaxed model is the same, except the coefficients of C_j variables. When λ_{jk} values are known, the relaxed model with makespan objective decomposes into two subproblems, one in assignment and one in cycle time variables, as follows:

Subproblem 1:

$$LR_1(z) = \text{Min} \sum_{j,k \in JK} \lambda_{jk} \sum_{r=1}^{R} p_{rk} X_{r(j-k+1)}$$

subject to: (1), (2) and

$$X_{r\,n} \leq 1 \qquad r = 1,..., R \qquad n = 1,..., N \tag{4'}$$

Constraint set (4) is replaced by (4'), since the resulting subproblem is a network problem. This subproblem can be solved by network simplex.

Subproblem 2:

$$LR_2(z) = \text{Min} \sum_{j=1}^{M+N+1} C_j - \sum_{j,k \in JK} \lambda_{jk} C_j$$

subject to: (5) and

$$C_j \leq \underset{r,k}{Max}\{p_{rk}\} \quad r = 1,..., R \qquad j,k \in JK=\begin{Bmatrix} j=1,...,M-1 & k=1,...,j \\ j=M,...,N & k=1,...,M \\ j=N+1,...,N+M-1 & k=j-N+1,...,M \end{Bmatrix} \tag{6}$$

Subproblem 2 is a simple unconstrained linear programming model with upper and lower bounds on variables, and can be solved easily by inspection. Constraint set (6) is a set of valid cuts for the subproblem.

After the relaxed problem is solved (to get a lower bound) by solving the two subproblems, a feasible solution (an upper bound) is found using the optimal solution X_{rn}* of Subproblem 1, and computing C_j values for each cycle by means of the relaxed constraint set (3).

We utilize subgradient optimization procedure by Fisher (1981) for solving the Lagrangean dual problem, which can be defined as $LD(z) = \underset{\lambda \geq 0}{\text{Max}}\{LR(z)\}$.

4. Computational Experience

A computational experiment is designed to test the efficiencies of the developed approximation procedure. The results show that our algorithm is very efficient especially for large problem instances for both objectives. Table 1 summarizes the performance of the algorithm on two problems for different number of jobs.

Table 1. Summary of the results

		CPU Time (sec.)		% Deviation from best LB	
Objective	**N**	**Average**	**Maximum**	**Average**	**Maximum**
	30	28.8	43	1.67%	8.78%
	60	37.4	50	4.75%	13.49%
Makespan	120	84.2	243	6.36%	21.44%
	30	7.8	10	2.34%	6.09%
	60	15.3	28	5.67%	15.10%
Total Flow Time	120	68.5	240	8.46%	22.43%

5. Conclusions

In this study, we studied the mixed model flow line sequencing problem (MMS) with makespan and total flow time objectives. We presented a Lagrangean Relaxation-based approximation algorithm especially for the large size instances of the problems.

We have tested our algorithm using computational experimentation. The algorithm has an outstanding performance especially for the makespan objective. We also observe that our approximation procedure generates high quality solutions in very small computation times for large instances, therefore it may be of great use when one is interested in nice and quick solutions rather than a guarantee of optimality.

References

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