OPTIMAL CONTROL OF A DEDICATED AND A FLEXIBLE SERVER AND THE VALUE OF FLEXIBILITY

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Abstract: In this paper we consider a production environment consisting of two machines and two products. Machine 1 is a dedicated server, which is capable of producing product 1 only. Machine 2 is a flexible server that can produce both product 1 and product 2. Both product 1 and product 2 are produced to stock. We characterize the form of the optimal policies that maximize the average profit per unit time for machine 1 and machine 2 by modeling the system as a Markov Decision Process. In a numerical study, we identify the situations when it may be beneficial to invest in a dedicated machine-flexible machine system instead of a fully dedicated or a flexible machine production system.

1. Introduction

In today's manufacturing environment most of the factories invest in both dedicated and flexible machines to combine the effectiveness of high volume production with competitive advantage of low volume, wide product variety. For example, in the plastics industry a typical investment strategy is to invest in many dedicated molding machines that produce only one product type in high volumes, and several molding machines that are capable of producing different sizes of parts and fittings with short switchover times.

There is little work on optimal scheduling of multiple job types on multiple machines when several servers are flexible and the others are dedicated. Farrar (1993) considers a two-stage system with a dedicated server at each queue and a flexible server serving both queues. He shows that the optimal policy can be characterized by a switching curve. Dueanyas *et al* (1998) consider the same two-stage production system with a single flexible server. Hopp and Van Oyen (2003) give a recent survey on the control of flexible servers.

2. Problem Formulation

We consider a production environment consisting of 2 parallel machines. The dedicated server (machine 1) produces only product 1 and the flexible server (machine 2) is capable of producing both product 1 and product 2. Both product 1 and 2 are produced to stock. The dedicated machine produces product 1 and the flexible machine produces product 2. We will call machine 1 and machine 2 as MTS1 and MTS2 machine respectively. The processing times at machine 1 and 2 are independent and exponentially distributed with

rates λ_1, and λ_2, respectively. Customer order requests for product 1 and 2 occur according to a Poisson process with rates, μ_1, and ¸μ_2, respectively. Each unit of product 1 and product 2 demand earns a revenue p_1 and p_2, and removes one unit of a product from the inventory. We assume that demand that is not satisfied from inventory is lost Inventory holding costs are incurred at a rate of h_i , *i* = 1*;* 2 per unit per

unit time for each unit of product *i* in stock. Our objective is to find an optimal policy π^i for each machine *i*, *i* = 1*;* 2 that consists of decisions d_j^i for each decision epoch *j*, $\pi^i = (d_1^i, d_2^i, ...)$ that maximizes the average profit earned per unit time over an infinite horizon by controlling the production of type 1 and type 2 products. In particular, at any decision epoch we would like to control the dedicated machine (machine 1) and decide whether it should continue to produce additional units of product 1 and for the flexible machine (machine 2) decide whether to stay idle, if not, whether to produce product 1 or product 2. We formulate this problem as a Markov Decision Process to characterize the form of the optimal policy. The decision epochs and the states in MDP are defined as follows.

- *Decision epochs*: (1) Production completion epochs at machine 1, and production completion

epochs at machine 2. (2) Product 1 and product 2 demand arrival epochs. We assume that the setup time and setup costs are negligible when the flexible machine switches from producing one type of product to another. Furthermore, we assume that the jobs at the flexible machine can be preempted.

- *States* of the production system at a decision epoch is represented by a vector (n_1, n_2) where

n_1 is the number of product 1 in the system, including the one being processed. Similarly n_2 ¸ represent the number of type 2 products in the system

- *Actions* at a machine 1 decision epoch are (1) Produce a type 1 product, (2) Stay idle, and

the actions at a machine 2 decision epoch are (1) Produce a type 1 product, (2) Produce a type 2 product, or (3) Stay idle.
After uniformazing the corresponding continuous time Markov Chain, the optimality equation for the Markov decision process with the objective of maximizing the average profit per transition is

$$g + v(n_1, n_2) = \left\{ \begin{array}{l} \frac{1}{\Lambda} - h_1 n_1 - h_2 n_2 + \lambda_1 (p_1 + v(n_1 - 1, n_2).I_{(n_1 > 0)} + v(0, n_2).I_{(n_1 = 0)} \\ \qquad + \mu_1 \max\{ v(n_1 + 1, n_2), v(n_1, n_2) \} \\ \qquad + \lambda_2 (p_2 + v(n_1, n_2 - 1).I_{(n_2 > 0)} + v(n_1, 0).I_{(n_2 = 0)}) \\ \qquad + \mu_2 \max\{ v(n_1 + 1, n_2), v(n_1, n_2 + 1), v(n_1, n_2) \} \end{array} \right\} \quad (1)$$

where $I_{(.)}$ denotes the indicator function and $\Lambda = \lambda_1 + \lambda_2 + \mu_1 + \mu_2$

3 .Characterization of the Optimal Policy

The structure of the optimal policy when both product 1 and product 2 are produced according to a make-to-stock regime is given by the following theorem.

Theorem 1

The optimal production policy for the MTS1 machine (the dedicated machine) is defined by a production curve $f(n_2)$ *such that for* $n_1 < f(n_2)$ *the optimal policy is to produce. Otherwise, it is optimal to stay idle. Furthermore,* $f(n_2)$ *is nonincreasing in* n_2. *The optimal production policy for the MTS2 (the flexible machine) is defined by functions* $g_1(n_2)$ *and* $g_2(n_2)$. *If* $n_1 > g_1(n_2)$ *then do not produce. If not, then produce type 1 product if* $n_1 < g_1(n_2)$. *Otherwise, produce product type 2. Furthermore,* $g_1(n_2)$ *is nonincreasing in* n_2 *and* $g_2(n_2)$ *is nondecreasing in* n_2.

The necessary and sufficient conditions for theorem 1 are as follows:

(i) $v(n_1; n_2) - v(n_1; n_2 + 1) \uparrow n_1$

(ii) $v(n_1; n_2 + 1) - v(n_1 + 1; n_2) \uparrow n_1$

(iii) $v(n_1 + 1; n_2) - v(n_1; n_2 + 1) \uparrow n_2$

for all n_1, n_2. The proof is based on eight lemmas, which we don't include here. Figure 1 shows the form of the optimal policies at machine 1 and machine 2.

4. Numerical Study and Conclusion

We performed a numerical study to quantify the value of flexibility and gain insights on when to invest on a one dedicated one flexible machine system. We studied the following three scenarios under the optimal scheduling policies. In scenario (a), machine 2 is dedicated, and machine 1 is the flexible machine. In scenario (b) machine 2 is the flexible machine and machine 1 is the dedicated machine. Finally in the third scenario, (scenario c) both machine 1 and machine 2 are dedicated machines. In our numerical study, we assumed that the cost of dedicated capacity is \$ 0.1 per unit. We varied the cost of flexible capacity per unit as 105 %, 110 %, 115 %, 150 %, and 200 % of the cost of dedicated capacity per unit. We investigated the effect of production system parameters on the value of flexibility and capacity investment decisions for each of these five examples. The production system parameters we considered are product 1 and product 2 order arrival rates, unit prices, dedicated and flexible machine production rates. In order to compare these scenarios we define “the value of flexibility" as

$$\% \text{ Value of flexibility} = \frac{\max\{R_{\text{mts1}}, R_{\text{mts2}}\}}{R_{\text{dedicated}}} \text{x}100$$

where R_{mts1}, and R_{mts2} are the average profit per unit time in scenario (a) and scenario (b), respectively. $R_{\text{dedicated}}$ is the average profit per unit time in scenario (c).

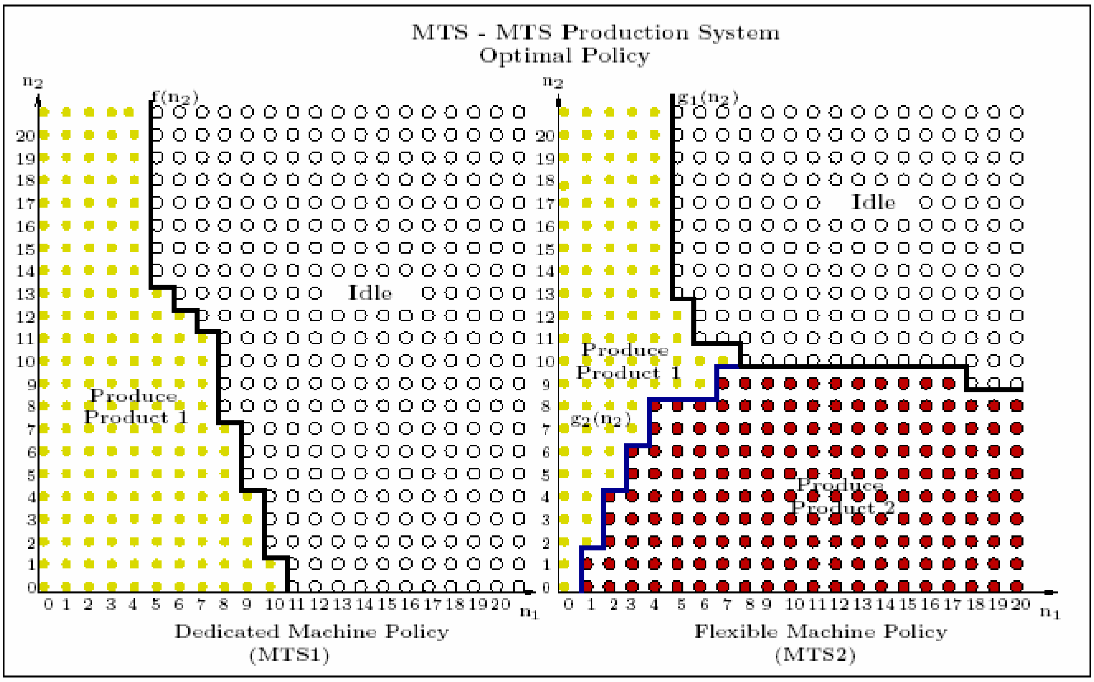

Figure 1: MTS-MTS System Optimal Policy when $\lambda_1 = 1.0$, $\lambda_2 = 3.0$, $\mu_1 = 0.5$, $\mu_2 = 5.0$, $p_1 = 2.0$, $p_2 = 1.0$, $h_1 = 0.001$, $h_2 = 0.005$

Our numerical study showed that the value of flexibility could be as much as 8% when the flexible capacity cost is low, and the value of flexibility decreases as the flexible capacity cost and/or product 2 demand rate
increases. According to the numerical study, the one dedicated one flexible system beneficial when the flexible capacity *cost* is moderate and the value of flexibility in the system we consider is at around 4%

References

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