# A TIME PARTITIONING HEURISTIC FOR THE SINGLE-LEVEL MULTI-ITEM CAPACITATED LOT-SIZING PROBLEM

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**Abstract:** In this study, we investigate a single-level multi-item multi-resource capacitated lot-sizing problem, which we encountered in the gearbox manufacturing plant of a major automotive manufacturer. The model is formulated as a mixed integer program for multiple items with independent demand and multiple shared resources with capacity constraints. We extend and analyze an effective partitioning heuristic.

***Keywords****: Capacitated Lot-Sizing Problem, Production Planning, Heuristics, Partitioning*

## 1. Introduction

The new competitive market environment forces the manufacturing firms to offer variety in their products. With increased product variety, production management has also become more complex. Many different products must be produced in the same time period and products should be manufactured in smaller lots. These small lot sizes and large number of products make the production planning more difficult. On the other hand, due to the existence of setups, producing large lot sizes are desired in order to cover demand over a number of future periods without backlogging. Although producing large batches will reduce the setup costs, this also increases inventory, which forms a tradeoff between the setup and inventory costs. Given the presence of this tradeoff, lot sizing decisions give rise to the problem of identifying when and how much of a product to produce at each period such that setup, production and holding costs are minimized. Making the right decisions in lot sizing will affect directly the system performance and its productivity, which are important for a manufacturing firm's ability to compete in the market. Therefore, developing and improving solution procedures for lot sizing problems is very important.

In this study, we investigate a capacitated lot sizing problem (CLSP), which we encountered in the gearbox manufacturing department of a major automotive company. The problem is to determine production lot sizes of different gearbox models in an environment where machine and labor capacities are restrictive. Gearbox has a single level bill-of-material. Hence, the problem is formulated as a multi-item multi-resource single-level CLSP.

The CLSP has been extensively studied in the literature. Some of the recent work includes Stadtler (2000), Miller *et al*. (2000a), Miller et al. (2000b), Gopalakrishnan *et al*. (2001), Stadtler (2003), Suerie and Stadtler (2003) and Federgruen *et al.* (2003). A recent survey can be found in Karimi *et al*. (2003). In our study, we extend the model and the partitioning heuristic investigated by Federgruen *et al.* (2003) to include capacity constraints of multiple shared resources.

## 2. The Single-Level Multi-Item Model with Multiple Resources

In this section, we provide the mixed integer formulation BM ($\alpha, \beta$) of the single level multi-item CLSP with multiple resources. We introduce parameters $\alpha$ and $\beta$ to facilitate our introduction to the partition heuristic, where they represent the indices of the first and the last period between which the integrality constraints are imposed.

Parameters of model BM ($\alpha, \beta$) are defined as follows:
$i$ = item index;
$N$ = number of items. $i = 1\ldots N$;
$t$ = period index;
$T$ = number of planning periods. $t = 1\ldots T$;
$j$ = resource index;
$J$ = number of resources. $j = 1\ldots J$;
$c_{it}$ = unit variable production cost of item $i$ produced in period $t$;
$h_{it}$ = cost of carrying a unit of inventory of item $i$ at the end of period $t$;
$q_{ij}$ = amount of resource $j$ required per unit of production of item $I$;
$D_{it}$= demand for item $i$ in period $t$;
$G_j$ = the group of items using the resource $j$;
$M$ = a large constant;

$S_{it}$ = setup cost incurred if item $i$ is produced in period $t;$
$C_{jt}$ = available capacity of resource $j$ in period $t;$
$O_{it}$ = outsourcing cost of item $i$ in period $t;$
$Ov_{jt}$ = overtime cost of resource $j$ in period $t;$

$$Y_{it} = \begin{cases} 1 & \text{if a setup exists for item i in period t} < \alpha; \\ 0 & \text{otherwise.} \end{cases}$$

Decision variables of model BM ($\alpha, \beta$) are defined as follows:

$X_{it}$ = production (lot size) of item $i$ in period $t;$
$I_{it}$ = ending inventory of item $i$ in period $t;$
$Q_{it}$ = amount of outsourced item $i$ in period $t;$
$C^0_{jt}$ = overtime capacity of resource $j$ in period $t;$

$$Y_{it} = \begin{cases} 1 & \text{if for t} >= \alpha \quad X_{it} > 0; \\ 0 & \text{otherwise.} \end{cases}$$

Finally, we need to define parameter $I^c_{jt}$ as the minimum ending capacity stock of resource $j$ at the end of period $t$ so that a feasible production plan exists for periods $t$+1, ..., $T$. Capacity stock indicates the capacity of the resource used to make production for following periods' demands. Initial minimum capacity stock levels are computed using the following formula:

$$I^c_{jt} = \Big(\sum_{i \in G_j} q_{ij} D_{it} - C_{jt} + I^c_{j(t+1)}\Big)^+,\ t = 1, ..., T\text{-}1, \text{ where } I^c_{jT} = 0.$$

Note that, as defined in this formula, $I^c_{jt}$ is a lower bound on the capacity that needs to be used to satisfy demand in the succeding periods to guarantee feasibility. In the proposed partition heuristic, we may set $I^c_{jt}$ to higher values to force partial model to produce more inventory in the earlier periods.

We now can formally define BM ($\alpha, \beta$) as

$$\text{Minimize } Z = \sum_{t=1}^{T}\left[\sum_{i=1}^{N}(\text{c}_{it}X_{it} + h_{it}I_{it} + O_{it}Q_{it} + S_{it}Y_{it}) + \sum_{j=1}^{J} Ov_{jt}C^0{}_{jt}\right]$$

Subject to

$$I_{j(t-1)} + X_{it} + Q_{it} = I_{it} + D_{it} \qquad i = 1, ..., N \text{ and } t = 1, ..., \beta$$

$$X_{it} \le MY_{it} \qquad i = 1, ..., N \text{ and } t = 1, ..., \beta$$

$$\sum_{i \in G_j} q_{ij} I_{it} \ge I^c_{jt} \qquad j = 1, ..., J \text{ and } t = 1,..., \beta$$

$$\sum_{i=1}^{N} q_{ij} X_{it} < C_{jt} + C^0_{jt} \qquad j = 1, ..., J \text{ and } t = 1,..., \beta$$

$$X_{it} \ge 0,\ I_{it} \ge 0 \qquad i = 1, ..., N \text{ and } t = 1, ..., \beta$$

$$Y_{it} \in \{0,1\} \qquad i = 1, ..., N \text{ and } t = \alpha, ..., \beta.$$

Note that BM (1, $T$) is the complete single-level, multi-item, multi-resource CLSP formulation.

## 3. The Extended Time Partitioning Heuristic

Partition heuristic is based on two parameters: $p$, partition length, and $s$, slide length. At each iteration $k = 0 \ldots (T - p) / s$, model BM ($\alpha_k = sk + 1, \beta_k = sk + p$) is solved, where values of parameters $Y_{it}$, for $t < \alpha$, are set to their optimum values calculated in the previous iterations. The main idea behind the partition heuristic is to keep the number of binary variables in any instance of BM, which is $Np$, independent of $T$.

We extend the partition heuristic by utilizing the following observation: At iteration $k$, in an optimum solution to BM ($\alpha_k, \beta_k$), ending inventories of items will be set so that

$$\sum_{i \in G_j} q_{ij} I_{i\beta_k} = I^c_{j\beta_k}, \text{ for } j = 1, ..., J,$$

since there is no incentive to hold more inventory than required to attain the minimum resource capacity stock at the last period. Hence, setup decisions are also effected by this end-of-planning-period phenomenon. However, at iteration $k + 1$, since $X_{it}$, for $t < \alpha_{k+1}$, are still decision variables, their values may increase, which indicates that more inventory is accumulated in the earlier periods than it was planned for in the $k$th iteration. The main idea in the extension to the partition heuristic is that, based on the new inventory accumulation decisions on the $(k + 1)$st iteration, we can re-formulate the model in the $k$th iteration to force more ending item inventory.

Let $I^{k+1}_{it}$ be the optimum inventory of item $i$ in period $t$ at iteration $k + 1$. After solving BM ($\alpha_{k+1}$, $\beta_{k+1}$), modify the resource capacity stock definition as given below, and reformulate and solve BM ($\alpha_k$, $\beta_k$).

$$I^c_{j\beta_k} = \sum_{i \in G_j} q_{ij} I^{k+1}{}_{i\beta_k} \qquad j = 1, ..., J.$$

The extended version has the opportunity to correct previous setup decisions based on new information obtained during iterations, without increasing the number of binary variables of BM models.

## References


**Federgruen, A., M. Tzur, and J. Meissner,** Progressive Interval Heuristics for Multi-Item Capacitated Lot Sizing Problems, submitted, 2003.

**Gopalakrishnan, M. , K.Ding, J.-M.Bourjolly, and S.Mohan,** A tabu-search heuristic for the capacitated lot-sizing problem with set-up carryover, *Management Science*, Vol.47, pp. 851–863, 2001.

**Karimi, B., Fatemi Ghomi,S.M.T., and Wilson, J.M.,** The capacitated lot sizing problem: a review of models and algorithms, *The International Journal of Management Science*, Vol.31, pp. 365-378, 2003.

**Miller, A., G.L.Nemhauser, and M.W.P.Savelsbergh,** On the polyhedral structure of a multi-item capacitated lot-sizing problems with set-up times by branch-and-cut, *CORE DP 2000/52*, Universit ´eCatholique de Louvain, Belgium, 2000a.

**Miller, A., G.L.Nemhauser, and M.W.P.Savelsbergh,** Solving multi-item capacitated lot-sizing problems with setup times by branch-and-cut, *CORE discussion paper*, No.00/39, Louvain-la-Neuve, Belgium**,** 2000b.

**Stadtler, H.,** Improved rolling schedules for the dynamic single level lot-sizing problem, *Management Science,* Vol.46, pp. 318–326, 2000.

**Stadtler, H.,** Multilevel Lot Sizing with Setup Times and Multiple Constrained Resources: Internally Rolling Schedules with Lot-Sizing Windows, *Operations Research,* Vol. 51, pp. 487-502, 2003.

**Suerie, C., H.Stadtler,** The Capacitated Lot-Sizing Problem with Linked Lot Sizes, *Management Science,* Vol. 49, pp. 1039-1054, 2003.