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**A REVENUE OPPORTUNITY MODEL FOR AIRLINE INDUSTRIES FOR THE  
DEREGULATED GLOBAL MARKET**

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**ABSTRACT**

This study analyzes yield management, that is, how to sell a finite inventory of perishable assets so as to maximize revenues. Here the assets are seats on a scheduled flight. We consider a two-class dynamic model, dealing with an opportunity to capture additional revenues by offering an upgrade to discount fare customers. We also extend the model to include overbooking and no-shows. The nature of the airline industry, especially for three decades is discussed and a brief summary of the future work is given.

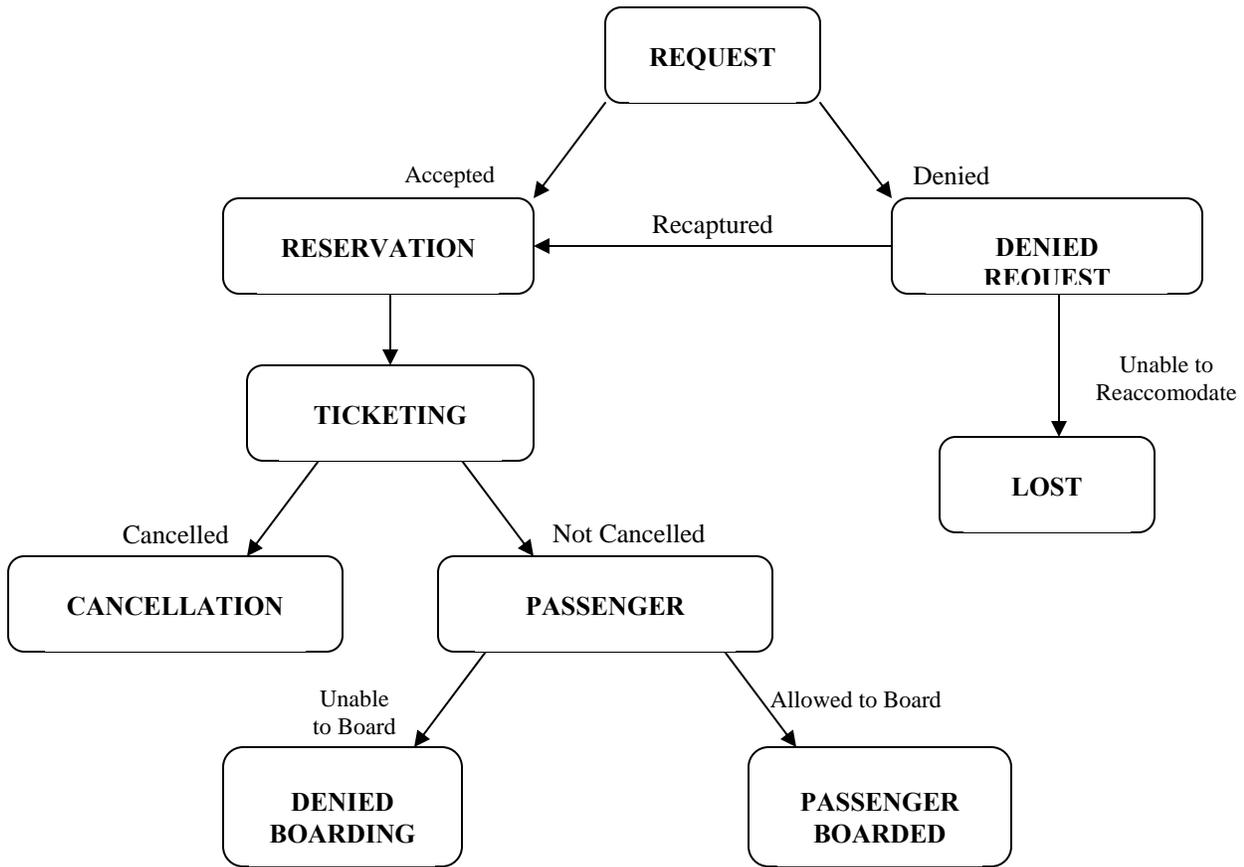
## I. INTRODUCTION

In the market place, service/product providers often use some limited resources to satisfy different classes of demands. This practice leads to question of how to manage the selling process of limited resources to maximize the total revenue. Yield management (YM) models addressing this issue are defined as multi-class models. In the literature, the multi-class models are commonly referred to airline seat allocation models, since most of these models have been constructed for identifying effective decision rules for airlines, which routinely book multiple fare classes into a common seating pool in an aircraft.

Airlines are able to change their pricing structures taking into account the fundamental differences between leisure travelers and business travelers. In general, business travelers are time sensitive and tend to make reservations closer to the departure date. On the other hand, leisure travelers are price sensitive and book well before the departure date. In order to protect seats for high-fare passengers, airlines need to limit seats availability to early booking low fare passengers. In other words, seats that are available for sale to a particular booking class are also available to bookings in any higher fare booking class, but not the reverse. This process is called nesting.

There are many researches done in seat inventory control or YM but none of them suggests a perfect modeling of revenue maximization, in terms of being realistic and at the same time being practical. There are a lot of variables to model and execution time of these models are long and they are not convenient to use in a reservation process of an airline. In other words, there is a trade of between being realistic and being practical in terms of modeling. Forecasts of passengers bookings by Origin & Destination, fare class and number of days before departure, shipment weights, equipment capacity, traffic, transaction volume etc are input to optimization and analysis modules of math programming, simulation, tabu search, heuristics, constraint logic programming, or network algorithms to make recommendations about YM availability, prices, routing plans, marketing decisions, markets to serve, hardware upgrades, manpower plans etc. All the data are used as input for Yield Management systems, which is driven by forecasting and optimization to control inventory. For forecasting demand, no-show and cancellation data are used. For optimization, overbooking, discount allocations and group evaluations data are used.

Figure 1 illustrates the entire booking process clearly:



**Figure 1** Booking Process

The remainder of the paper is organized as follows. In section 2, earlier works in revenue management and airline industry are summarized. The objective of the study and the problem formulations are described in sections 3. Finally, conclusion remarks and future works are provided in section 4.

## II. LITERATURE REVIEW

In general, the previous work on airline yield management can be categorized as belonging to one of two basic model types, static or dynamic. In a static model, the booking period is to set a booking limit for every booking class at the start of the booking process. A weakness of this approach is its inability to consider the actual current booking status during the process. However it can handle large problems and can address the multiple flight-leg problems. A dynamic model sets the booking limit for each booking class according to the actual bookings throughout the entire booking process. In terms of accuracy, the dynamic models can be better than the static models. A weakness of the dynamic approach is that it is computationally intensive.

Belobaba (1987 a, b, 1989) developed an Expected Marginal Seat Revenue (EMSR) approach to find an approximation to an optimal policy for the single leg, multi-fare problem. He developed the idea of Littlewood (1972) as that was for two fare classes only. In order to determine when to turn away class  $i$  booking requests, he solves the two-fare problem for fares 1 and  $i$ , 2 and  $i$ , ...,  $i-1$  and  $i$  to obtain  $S_1, S_2, \dots, S_{i-1}$  where  $S_i$  is the empty seat reserved for  $i^{\text{th}}$  booking class and  $f_i < f_{i-1}$  ( $f$  is the class fare in dollars). Class  $i$  booking requests are rejected if the number of empty seats is  $S_1 + S_2 + \dots + S_{i-1}$  or less, and accepted otherwise. He finds that while the optimal policy often differed significantly from that of the EMSR model, the expected revenue from the optimal policy is extremely close to that of the EMSR policy. EMSR heuristic uses pair wise fare comparisons to quickly arrive at approximate booking limits. Because of its computational ease, his EMSR heuristic provides a natural alternative to the optimal policy.

Weatherford, Bodily and Pfeifer (1993) examine dynamic booking limits for two classes of passengers with sell-ups and overlapping dynamic arrival rates based on Belobaba's work. Independently, McGill (1988) and Curry (1990) developed models for the case where lower classes are booked first. They use continuous demand distributions. McGill's expressions for optimal booking limits are probability statements that require integration. Curry's optimal booking limits are expressed in terms of a convolution integral. He examines booking limits for a network of flights. He divides each origin & destination itinerary into one or more fare class-

nests each of which contains at least one fare class. Curry shows that the revenue received from each nest is a concave function of the space allotted to it. He uses linear programming to allocate seats to the individual fare class-nests by approximating the nest revenues by piecewise linear functions. His formulation does not allow seats to be swapped among the various nests.

Wollmer (1992), Brumell and McGill (1993), and Robinson (1995) investigated the single-leg problem with multiple fare classes. They showed that Belobaba's heuristic is sub optimal. They developed procedures to find the optimal booking policy under the assumption that the probability of filling the plane is known. Liang (1999) proposed a continuous-time dynamic yield management model and showed that a threshold control policy is optimal. The control policy is for an arbitrary number of fare classes and arbitrary booking curves. Zhao and Zheng (1998) proved that a similar threshold control policy is optimal for a more general airline seat allocation model that allows diversion/upgrade and no-shows.

In particular, Subramanian, Stidham, and Lautenbacher (1999) present a model permitting cancellations, overbooking and discounting. They develop a discrete time, finite horizon Markov Decision Process (MDP), and solve by backward induction on the number of periods remaining before departure. Gallego (1996), Lee and Hersh (1993), Rothstein (1985) and Talluri and van Ryzin (1999) can be referred for overbooking policy and bid-price control. Lee and Hersh (1993) present a general model of booking limits for multiple fare classes and multi seat booking requests by subdividing time into sufficiently small intervals. On the other hand, van Slyke and Young (2000) study a time dependent finite horizon stochastic knapsack model. They characterize the optimal return function and the optimal acceptance strategy for this problem. For a comprehensive list of revenue management work, one can refer the survey paper of McGill and van Ryzin (1999).

However none of the researches mentioned above deals with the case of stochastic capacity in the future. All of them assume the capacity available is deterministic while the demand is uncertain. Wang and Regan (2002) propose a solution for the continuous stochastic dynamic yield management problem in which flight capacities are subject to change. They suggest aircraft assignments accordingly. The problem is divided into two periods. The result from the second

stage is used to derive the salvage function for the first period for determination of the optimal policy. They also claim that though only the simple case of changing capacity is considered, the method developed can be extended to a more general case where the capacity can be changed at multiple times and to multiple levels.

Pak, Dekker and Kindervater (2003) show how to incorporate the shifting capacity opportunity into a dynamic, network-base revenue management model. They use convertible seats for shifting business and economy class capacities. A row of these seats can be converted from economy class to business class seats and vice versa. When a row is converted from business to economy class, the number of seats in the row is increased and the width of each seat is decreased. It can be analyzed under dynamic capacity management.

### **III. MODEL DESCRIPTION AND PROBLEM FORMULATION**

#### **i. Objective of the study**

There has been an increasing demand for additional revenue opportunities of airlines in this highly competitive industry. In this study, a potential revenue opportunity is modeled with some simplifying assumptions that mentioned later in the text.

Overbooking is the practice of ticketing seats beyond the capacity of an aircraft to minimize the revenue lost caused by no-shows and to capture the additional revenue from the additional customers. However, when airlines sell more than available seats, there is always a possibility of denied boarding representing a cost figure. Airlines are usually divided into groups called cabins depending on the types of aircraft. Each cabin has different capacities and each seat in a cabin associated with different marginal revenues. In practice it is quite likely for the demand of economy cabin to exceed the cabin capacity in certain flights. For the same flights forecasted demand for the business or first cabin may be less than its capacity. When lower fare class tickets are oversold and at the same time marginal expected revenue of selling one more higher class ticket is less than marginal expected revenue of selling one more low fare class ticket, it is sensible to give higher class seats to lower class passengers by asking some additional value than their ticket price. If a passenger accepts this offer and gives extra money to pass higher-class seat (business or first), airline gets/captures additional revenue and also gets rid of some denied

boarding costs. In this study a theoretical reasoning is searched for this practical issue. A dynamic mathematical model is developed to seek a revenue opportunity of this case.

## **ii. Problem Formulation**

The dynamic multi-class model of Zhao (1999) is modified to capture additional revenue from discount fare customers who accepts the upgrading offer. This model assumes that demands for both classes arrive concurrently according to independent, non-homogenous Poisson processes. It is formulated as an optimal stopping problem to determine when to close lower fare class. This dynamic model is closely related with the static model proposed by Littlewood (1972), which is also known as protection level policy. The dynamic seat allocation model is extended to incorporate overbooking later.

In our model, there are different types of customers. Full fare customers, discount fare customers, customers that have already discount fare but also accept the offer of upgrade. One can consider those customers and discount fare customers as one type for simplicity. Arrival process of customers is Poisson and independent of each other. Also it is assumed that once closed, the discount fare is not allowed to reopen. In this work we demonstrate the booking process in three different scenarios. First, a general model in which no upgrading option is offered to economy cabin customers is given. Next we modified this model to incorporate upgrading option with an additional fare. Finally we included no-shows and overbooking possibilities to the model to capture the extra revenue.

Descriptions of some of the notations that are used in the formulation of problem are given below.

$C$ : Number of seats on a scheduled flight, which can be sold at two different fares (full and discount).

$R_i$ : Revenue gained from class  $i$  ticket. (Class 1 is the full fare class, and Class 2 is the discount fare class.

$t$ : Time left before departure.

$\lambda_i(t)$ : Arrival rate of class  $i$  customers.

$u(t)$ : The probability of upgrading a discount customer, arriving at time  $t$ .

$\lambda_i(t)$  and  $u(t)$  are assumed to be piece wise continuous with a finite number of discontinuous points.

$q$ : The probability of a booked customer showing up at departure time.

### **BASIC MODEL: No Upgrading Offer To Discount Fare Customers**

Let  $x(t)$  denotes the total number of seats that have been booked and  $y(t)$  denotes the number of seats left and  $y(t) = C - x(t)$  then the decision of whether to close discount fares should be based on  $y$  and  $t$ . If  $D_i(t, s)$ : the total demand for class  $i$  fare that occurs between  $t$  and  $s$ . ( $t > s$ )

then one can claimed that  $D_i(t, s)$  is a Poisson random variable with the mean  $\int_s^t \lambda_i(\tau) d\tau$ .

If  $R_s(y, t)$  is the expected revenue between  $[t, 0]$  under a given threshold (stopping rule)  $S$ , starting with  $y$  seats left at time  $t$  then the expected revenue between  $[t, 0]$  can be written as:

$R_s(y, t) = E(r_1 D_1(t, s) + r_2 D_2(t, s) + r_1 \min\{D_1(s, 0), y - D_1(t, s) - D_2(t, s)\})$  where  $s$  is the stopping time determined by the stopping rule  $S$ .

Let us denote the maximum expected revenue starting with  $y$  seats left at time  $t$  by  $f(y, t)$  and the expected revenue between  $[t, 0]$  with  $y$  seats left at time  $t$  with only class 1 is open and with class 2 is already closed by time  $t$  by  $g(y, t)$ . The optimal stopping rule now can be characterized by  $S^* = \{(t, y) \mid f(y, t) = g(y, t)\}$ . If  $(t, y) \notin S^*$ ,  $f(y, t) > g(y, t)$  then it is optimal to keep class 2 open. If only class 1 is open, then the expected revenues for  $y, y+1$  and zero seat left are  $g(y, t) = E(r_1 \min\{D_1(t, 0), y\})$ ,  $g(y+1, t) - g(y, t) = r_1 P\{D_1(t, 0) \geq y+1\}$  and  $g(0, t) = 0$ . At any given time  $t$  when class 2 is closed, there is no further decision to make. Since the demand of class 1 fare is Poisson, we have

$g(y, t) = \lambda_1(t) \delta t (r_1 + g(y-1, t - \delta t)) + (1 - \lambda_1(t) \delta t) g(y, t - \delta t)$ , where  $\delta t$  is the length of a small time interval  $[t, t - \delta t]$ . The first term in the right hand side of the equation is the expected total revenue if a ticket is sold during  $(t, t - \delta t)$ , and the second term is the total expected revenue otherwise. If we let  $\delta t$  to go towards 0, then the following derivation is obtained:

$\frac{\partial g(y,t)}{\partial t} = \lambda_1(t)(r_1 + g(y-1,t)) - \lambda_1(t)g(y,t)$ . If it is optimal to keep class 2 open at  $(y, t)$ , the

maximum expected revenue is estimated by the following dynamic programming equation:

$$f(y,t) = \lambda_1(t)\delta t(r_1 + f(y-1,t-\delta t)) + \lambda_2(t)\delta t(r_2 + f(y-1,t-\delta t)) + (1 - \lambda_1(t)\delta t - \lambda_2(t)\delta t)f(y,t-\delta t)$$

If  $\delta t$  approaches to 0, one can derivate this function with respect to time left before departure

and  $\frac{\partial f(y,t)}{\partial t} = \lambda_1(t)(r_1 + f(y-1,t)) + \lambda_2(t)(r_2 + f(y-1,t)) - (\lambda_1(t) + \lambda_2(t))f(y,t)$  is obtained.

The above equation only holds when  $(t,y) \notin S^*$ . When  $(t,y) \in S^*$  is true then we have

$f(y,t) = g(y,t)$  where  $f(y,t)$  and  $g(y,t)$  are continuous in  $t$ .

*The static threshold value is found according to following condition:*

Keep the discount fare class open if and only if  $r_2 > r_1 P\{D_1(t,0) \geq y\}$  [Littlewood (1972), Belobaba (1989) and Weatherford (1992)]. Left hand side (LHS) of this equation is the revenue from selling a seat at the discount fare and right hand side (RHS) of it is the opportunity cost of the last seat. Finally for the dynamic model, this rule becomes  $\{(t,y) | r_2 \leq r_1 P\{D_1(t,0) \geq y\}, y \in \{0,1,\dots,C\}\}$ .

### **GENERAL MODEL: Offer to Discount Fare Customers**

Now let us denote the demand rate for class 1 seats when class 2 is closed by  $\lambda(t)$  (Demand of both class 1 customers and customers that have initially discount fares but accept the upgrading offer). For two-class problem, demand rate is found as

$\lambda(t) = \lambda_1(t) + u(t)\lambda_2(t)$ , where  $u(t)$  is the probability of upgrading a discount customer. Total

demand in  $[t, 0)$  for the full fare is given by  $D(t,0)$  and it is Poisson with mean  $\int_s^t \lambda(\tau) d\tau$ . Now

the expected revenue  $g(y,t)$  satisfies the following equation:

$$g(y,t) = \lambda(t)\delta t(r_1 + g(y-1,t-\delta t)) + (1 - \lambda(t)\delta t)g(y,t-\delta t) \text{ and}$$

$$\frac{\partial g(y,t)}{\partial t} = \lambda(t)(r_1 + g(y-1,t) - g(y,t))$$

Modified Littlewood condition is:

$$r_2 > u(t)r_1 + (1-u(t))r_1P\{D(t,0) \geq y\}$$

Left hand side of this equation is the revenue if the request is satisfied. The right hand side of it is the opportunity cost of a seat when the request is denied. The expected revenue from this seat is  $u(t)r_1$  if the customer accepts the offer to upgrade, otherwise the expected revenue is  $(1-u(t))r_1P\{D(t,0) \geq y\}$ . Modified Littlewood condition can also be written as:

$$r_2 > u(t)r_1 + (1-u(t))(g(y,t) - g(y-1,t))$$

### **GENERAL MODEL: Offer to Discount Fare Customers Incorporating No-Shows And Overbooking**

No shows are referred to as those booked customers who fail to show up at departure. Airlines overbook to compensate for no-shows. It is assumed that customers of different classes have similar no-show behavior. Otherwise, the model would have to be more complex although in general it is not a very realistic assumption.

In the formulation of the opportunistic revenue model incorporating the no-shows and overbooking, expected revenue from the sale of class  $i$  is redefined and denoted by  $r_i$ . Net revenue obtained from the sale of class  $i$  is given by:

$$\pi_i = f_i - c_v$$

where  $f_i$  is the class fare and  $c_v$  is the variable cost.

If  $p_i$  is the penalty paid by a no-show customer after refund and  $q$  is the probability of show-up for booked passengers then one can find the expected as:  $r_i = q\pi_i + (1-q)p_i$ . Now let us denote

the total number of tickets that have been sold before the departure by  $X$  and the number of passengers who show up at departure by  $Z(x)$  then the number of passengers who would be

denied from boarding is given by  $\varpi = \begin{cases} Z(x) - C & \text{if } Z(x) > C \\ 0 & \text{otherwise} \end{cases}$  and  $Z(x)$  is binomial with  $B(x,$

$q)$ .

If the total cost incurred for denying  $n$  passengers from boarding given by  $\Phi(n)$  then the expected cost incurred for denied boarding (over sale)  $h(x)$  is  $E[\Phi(\varpi)]$ . Since  $\Phi(x)$  is assumed to be increasing and convex (because the marginal cost for denying a passenger from boarding increases with the number of denials)  $h(x)$  is also increasing and convex. The total expected revenue  $R$  is  $r_1 x_1 + r_2 x_2 - h(x_1 + x_2)$ . If  $x$  tickets have already been sold, the net marginal expected revenue from selling an extra ticket of class  $i$  is  $r_i - \Delta h(x)$ , where  $\Delta h(x) = h(x+1) - h(x)$ , is the expected marginal over-sale cost. A class  $i$  ticket should be sold if and only if  $r_i - \Delta h(x) > 0$ . So if  $L$  defines the following region,  $L = \max\{x \mid r_1 - \Delta h(x) > 0\}$  where  $L$  is the maximum number of tickets available for sale and  $x(t)$  is the total number of tickets sold up to time  $t$  then  $y(t) = L - x(t)$  that is the number of tickets left at time  $t$ , if originally there were  $L$  tickets printed. So the previous model is modified as replacing  $r_i$  by  $r_i - \Delta h(L - y)$ :

$$g(y, t) = E(r_1 \min\{D(t, 0), y\} - (h(L - y + \min\{D(t, 0), y\}) - h(L - y)))$$

$$\text{then } g(y, t) = E\left(\sum_{k=0}^{\min\{D(t, 0), y\}-1} (r_1 - \Delta h(L - y + k))\right)$$

One can claim that, for any  $t$ ,  $g(y, t)$  satisfies the following equation:

$$g(y+1, t) - g(y, t) = \left( \begin{array}{l} P\{D(t, 0) \geq y+1\}(r_1 - \Delta h(L - y - 1)) \\ + \\ \sum_{j=1}^y P(D(t, 0) = j)(\Delta h(L - y + j - 1) - \Delta h(L - y - 1)) \end{array} \right)$$

and  $g(0, t) = 0$ . The first term of the RHS is the net expected revenue of the  $(y+1)^{\text{st}}$  ticket. The second term is the reduced over sale cost due to the fact that starting with  $(y+1)$  tickets at  $t$ , instead of  $y$  tickets, that means one less ticket is sold. We can replace  $P\{D(t, 0) = j\}$  by

$$P\{D(t, 0) \geq j\} - P\{D(t, 0) \geq j+1\} \text{ so,}$$

$$g(y+1,t) - g(y,t) = \left( \begin{aligned} &P\{D(t,0) \geq y+1\}(r_1 - \Delta h(L-1)) + P(D(t,0) \geq 1)(\Delta h(L-y) - \Delta h(L-y-1)) \\ &+ \\ &\sum_{j=2}^y P(D(t,0) \geq j)(\Delta h(L-y+j-1) - \Delta h(L-y+j-2)) \end{aligned} \right)$$

*Modified Littlewood condition is:*

Keep the discount fare open if and only if

$$r_2 - \Delta h(L-y) > u(t)r_1 - \Delta h(L-y) + (1-u(t))(g(y,t) - g(y-1,t)) \text{ condition holds.}$$

The left hand side is the net expected marginal revenue from selling a class 2 ticket, and the right hand side is the marginal expected revenue if this ticket is held back to a full fare customer (discount fare customer before the offer in fact). In the presence of our offer for discount fare customers to upgrade, the threshold does not have to decrease over time.

#### **IV. FUTURE EXTENSIONS AND CONCLUSION**

As a result, the closure of the discount fare does not have to immediately follow a sale. The model is highly depend on the function,  $u(t)$ . So, further analysis can be done for the cases in which  $u(t)$  is decreasing or increasing or constant. And efficiency of model is assessed according to different results obtained from different cases.

In practice airlines tend to upgrade passengers from coach class to business class when coach capacity is less than demand and business capacity is higher than demand. We observe similar practices in other industries such as car rental companies, cruise lines and hotels. To the best of our knowledge, our model is the first formulize this business practice to capture this opportunistic revenue. Most desirable objectives for the airline industry would be to ensure greater stability in the marketplace and to improve profitability of the airline industry.

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